

Name: _____ Date: 10/13/25

Session 9 ☺

3 big things we want to look at:

- Law of Sines
- Law of Cosines
- Heron's Area Formula

★ Law of Sines
Case 1 - ASA

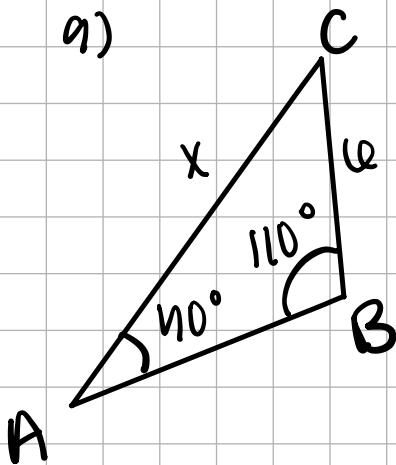
Case 2 - SSA
↑
ambiguous
case !!

★ Law of Cosines
Case 3 - SAS

Case 4 - SSS

1. Find the missing variable in each triangle using Law of Sines or Law of Cosines (Identify which law you're using first!)

a)

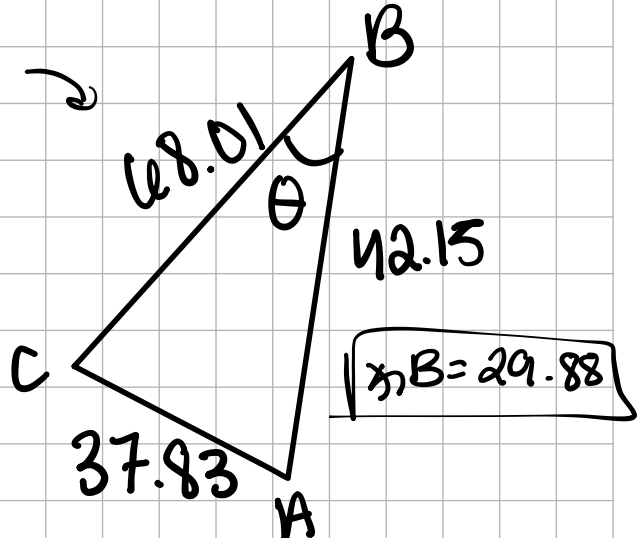


$$\sin 40^\circ \cdot \frac{6}{\sin 40^\circ} = \frac{x}{\sin 110^\circ} \cdot \sin 110^\circ$$

$x = 8.77$

b)

law of
cosines

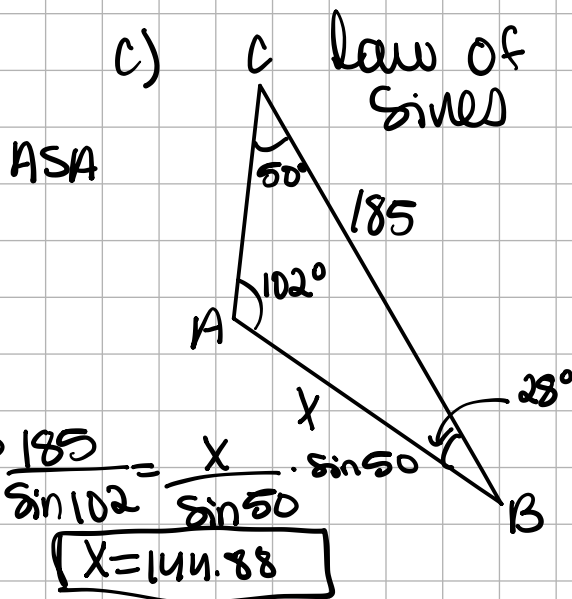


$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

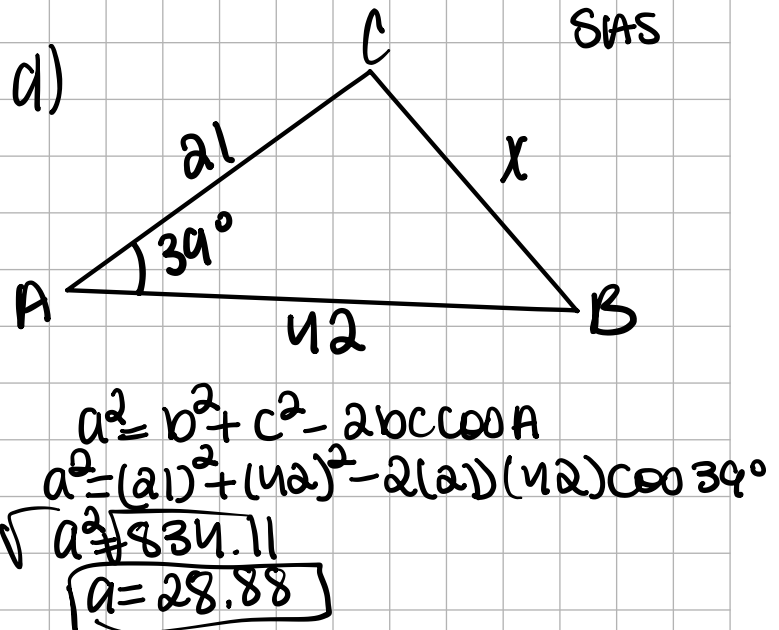
$$\cos B = \frac{(68.01)^2 + (42.15)^2 - (37.83)^2}{2(68.01)(42.15)} = 0.8670$$

$$\cos^{-1}(0.8470) = B$$

1. Continued



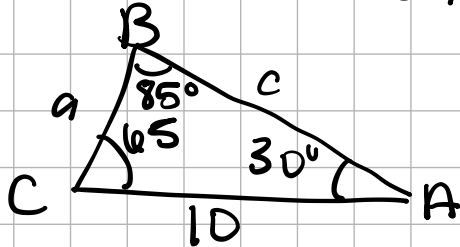
Law of Cosines



2. Solve for the missing variable, but now you draw your own triangle!

a) $\angle A = 30^\circ$, $\angle C = 65^\circ$, $b = 10$

ASA



we want:

$$a = 5.01$$

$$c = 9.09$$

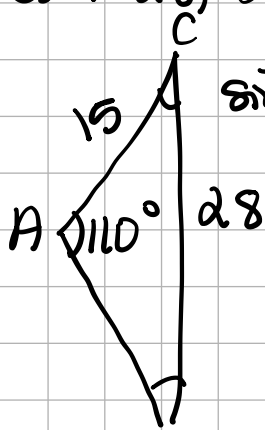
$$\angle B = 85^\circ$$

$$\sin 30^\circ \cdot \frac{10}{\sin 85^\circ} = \frac{a}{\sin 30^\circ} \quad \sin 65^\circ \cdot \frac{10}{\sin 85^\circ} = \frac{c}{\sin 65^\circ}$$

$$a = 5.01$$

b) $a = 28$, $b = 15$, $\angle A = 110^\circ$

SSA



$$\frac{c}{\sin 39.8^\circ} = \frac{28}{\sin 110^\circ}$$

$$15 \cdot \frac{\sin 110^\circ}{28} = \frac{\sin B}{15} \cdot 15$$

$$\sin B = 0.5034$$

$$\sin^{-1}(0.5034) = 30.22^\circ$$

$$180 - 30.22 = 149.78$$

$$180 - 30.22 - 149.78 = -79.78$$

only one!!

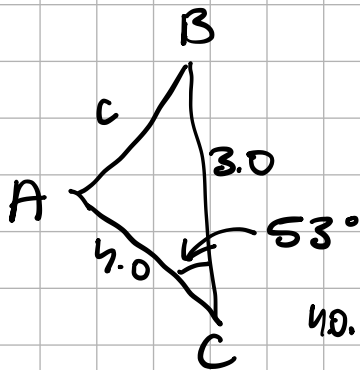
$c = 19.01$ Ambiguous case

$$\angle B = 30.22^\circ$$

$$\angle C = 39.8^\circ$$

$$c = 19.01$$

c) $a=3.0$, $b=4.0$, $\angle C=53^\circ$ $180-53-80.39=46.61$



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 3^2 + 4^2 - 2(3)(4) \cos 53^\circ$$

$$\sqrt{c^2} = \sqrt{10.554}$$

$$\boxed{c = 3.24}$$

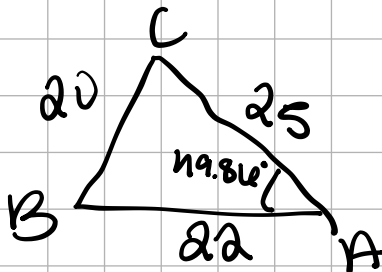
$$40. \frac{\sin 53}{3.24} = \frac{\sin B}{4.0} \cdot 4.0$$

$$\sin B = 0.98596$$

$$\sin^{-1}(0.98596)$$

$$\boxed{\angle B = 80.39^\circ}$$

d) $a=20$, $b=25$, $c=22$



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\boxed{\angle C = 57.252^\circ}$$

$$\frac{(25)^2 + (22)^2 - (20)^2}{2(25)(22)}$$

$$\cos A = 0.6445$$

$$\cos^{-1}(0.6445) = A$$

$$\boxed{\angle A = 49.848^\circ}$$

$$25. \frac{\sin 49.848}{20} = \frac{\sin B}{25} \cdot 25$$

$$\sin B = 0.9554$$

$$\sin^{-1}(0.9554) = \angle B = 72.88^\circ$$

3. Find the area of the triangle whose sides have the given lengths

a) $a=9$, $b=12$, $c=15$

$$s = \frac{9+12+15}{2} = 18$$

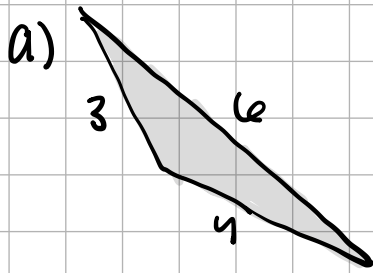
$$A = \sqrt{18(18-9)(18-12)(18-15)} = \boxed{54}$$

b) $a=7$, $b=8$, $c=9$

$$s = \frac{7+8+9}{2} = 12$$

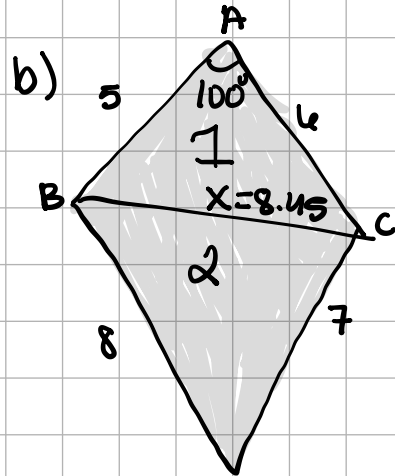
$$\sqrt{12(12-7)(12-8)(12-9)} = \boxed{24.83}$$

4. Find the area of the shaded figure



$$s = \frac{3+4+6}{2} = 6.5$$

$$\sqrt{6.5(6.5-3)(6.5-4)(6.5-6)} = \boxed{5.33}$$



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = (6)^2 + (5)^2 - 2(6)(5) \cos 100$$

$$\sqrt{a^2} = \sqrt{71.418}$$

$$a = 8.45$$

$$s = 9.725$$

$$\sqrt{9.725(9.725-5)(9.725-6)(9.725-8.45)}$$

↓

11.77

$$2s = 11.725$$

$$\sqrt{11.725(11.725-7)(11.725-8)(11.725-8.45)}$$

↓

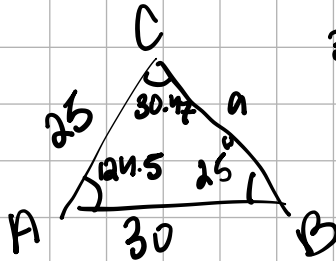
25.99

$$\boxed{11.77 + 25.99 = 110.77}$$

extra problem

ambiguous case

$$b=25, c=30, \angle B=25^\circ$$



$$30 \cdot \frac{\sin 25}{25} = \frac{\sin C}{30} \cdot 30$$

$$\sin C = 0.507$$

$$\sin^{-1}(0.507) = C$$

$$\angle C = 30.47^\circ$$

$$\angle A = 124.53$$

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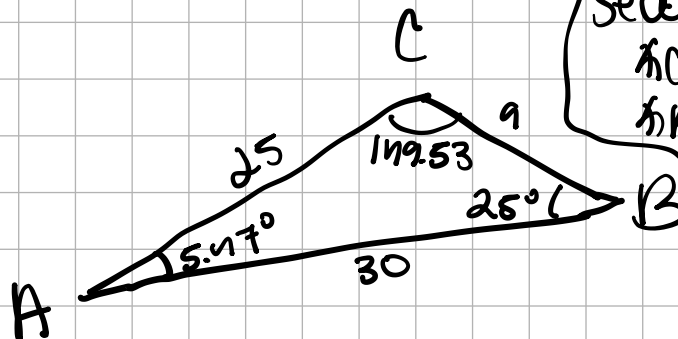
$$\frac{\sin 124.53 \cdot a}{\sin 124.53} = \frac{25}{\sin 25} \cdot \sin 124.53$$

$$a = 19.73$$

$$\#2 \quad 180 - 30.47 = 149.53$$

now we test !!

$$180 - 149.53 - 25 = 5.47^\circ$$



Second triangle

$$\angle C = 149.53$$

$$\angle A = 5.47$$

$$a = 5.64$$

$$\frac{\sin 5.47 \cdot a}{\sin 5.47} = \frac{25}{\sin 25} \cdot \sin 5.47$$

$$a = 5.64$$

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