Name: Answer Key Date: 11/29/16

I made this test prep with your test in mind, so you shouldn't need a calculator!

Test Prep 1

1. Simplify or factor

a) $\frac{8a^{10}b^7}{16a^3b}$ → $\frac{a^7b^6}{2}$

$\frac{a^{10}}{a^3} \to a^7$ $\frac{b^7}{b} \to b^6$

$\frac{8}{16} \to \frac{1}{2}$

b) $2x^2+16x-36$

$2(x^2+3x-18)$

18: 1 | 18, 2 | 9, 3 | 6

$2(x+6)(x-3)$

$(x+6)(x-3)$ Foiling to check

$x^2-3x+6x-18$

$2(x^2+3x-18)$

$2x^2+16x-36$

c) $x^4-7x^3+10x^2$ 10: 1 | 10, 2 | 5

$x^2(x^2-7x+10)$

$x^2(x-5)(x-2)$

You can take x^2 out of every term

d) $3x^2-12$

$3(x^2-4)$ ← difference of squares

$3(x+2)(x-2)$

2. Solve the rational equations and check for extraneous solutions

Hint: Find a LCD
it's always good to try and factor wherever you can!

a) $\frac{1}{x+5}+\frac{1}{x-5}=\frac{4}{x^2-25}$

$\frac{1}{x+5}+\frac{1}{x-5}=\frac{4}{(x+5)(x-5)}$

$(x-5)+(x+5)=4$

$\frac{(x+5)(x-5)}{1}\cdot\frac{1}{x+5}+\frac{1}{x-5}=\frac{4}{(x+5)(x-5)}\cdot\frac{(x+5)(x-5)}{1}$

$x-5+x+5=4 \rightarrow \frac{2x=4}{2}$ $\boxed{x=2}$

b) $\frac{3}{n+1}=\frac{5}{n-3}$

$\frac{(n-3)(n+1)}{1}\cdot\frac{3}{n+1}=\frac{5}{n-3}\cdot\frac{(n-3)(n+1)}{1}$

$3(n-3)=5(n+1)$

$3n-9=5n+5$
$-3n-5 \quad -3n-5$

$\frac{2n=-14}{2}$

$\boxed{n=-7}$

3. Solve these quadratics by completing the square

a) $x^2+4x+3=0$

$x^2+4x\ \underline{+4}=-3+4$

$4/2 \rightarrow 2^2=4$

$\sqrt{(x+2)^2}=\sqrt{1}$

$x+2=1$
$-2\ -2$
$\boxed{x=-1}$

$x+2=-1$
$-2\ -2$
$\boxed{x=-3}$

b) $2x^2-8x+9=0$

$2x^2-8x\ \underline{\quad}=-9$

$2(x^2-4x\ \underline{+4})=-9+\underline{2(4)}$

$-4/2 \rightarrow (-2)^2=4$

$2(x-2)^2=-9+8$

$\frac{2(x-2)^2=-1}{2}$

$\sqrt{(x-2)^2}=\sqrt{-1/2}$

$\boxed{(x-2)=\pm\frac{1}{\sqrt{2}}}$

I'm leaving it here

you have to multiply by what you took out

4. Find the difference quotient of

$f(x) = 3x+2$

difference quotient $= \frac{f(x+h)-f(x)}{h}$

$$\frac{3(x+h)+2-(3x+2)}{h}$$

after canceling like terms

$$\frac{3x+3h+2-3x-2}{h} \rightarrow \frac{3h}{h} \quad \frac{h(3)}{h} \rightarrow \boxed{3}$$

5. For the function $f(x)=(x-5)^2$ whose graph is shown below, find the net change and average rate of change between the following values

$x=1$ and $x=4$ (a, b)

net change is $f(b)-f(a)$

$(4-5)^2-(1-5)^2$

$(-1)^2-(-4)^2$

$1-16$

$\boxed{-15}$

average ROC is $\frac{f(b)-f(a)}{b-a}$

$\frac{-15}{4-1}$

$\frac{-15}{3}$

slope $\rightarrow \boxed{-5}$

6. Polynomial long division

a) Divide $(x^3+9x^2+23x+15)$ by $(x+1)$. Then, factor the result.

$$x+1 \overline{)\, x^3+9x^2+23x+15}$$

$$x^2+8x+15$$

$$-(x^3+x^2)$$

$$0+8x^2+23x$$

$$-(8x^2+8x)$$

$$15x+15$$

$$-(15x+15)$$

$$0$$

$x^2+8x+15$ $\quad$ 15

$(x+3)(x+5)$ $\quad$ $15 \cdot 1$, $3 \cdot 5$

$$(x+3)(x+5)(x+1)$$

7. Solve $e^{2x}-e^x-6=0$

let $u=e^x$

$u^2-u-6=0$ $\quad$ 6

$(u-3)(u+2)=0$ $\quad$ $6 \cdot 1$, $2 \cdot 3$

$u=3$ $\quad$ $u=-2$

$e^x=3$ $\quad$ $e^x=-2$ - not a solution

$\ln 3=x$ $\quad$ $\ln -2=x$

only answer

8. Evaluate the following rational functions

a) $\frac{2x^2-8x-24}{x^2-5x-6}$ $\quad \frac{2(x^2-4x-12)}{x^2-5x-6}$ $\quad \frac{2(x-6)(x+2)}{(x-6)(x+1)}$

① horizontal asymptote - powers are the same so the coefficients are the h.a. $2/1 = h.a = 2$ or $\boxed{y=2}$

② hole - $(x-6)$ cancels out on top & bottom

$x=6$

to find y-value → $\frac{2(x+2)}{(x+1)}$ $\quad \frac{2(6+2)}{(6+1)}$ $\quad \frac{16}{7}$

$\boxed{\text{hole at } (6, 16/7)}$

③ v.a. left-over in denominator $\boxed{x=-1}$

④ x-int left-over in numerator

x-int @ $\boxed{x=-2}$

b) $\frac{x^2+5x+4}{x-3}$ $\quad \frac{(x+4)(x+1)}{x-3}$

$$x-3 \overline{)\, x^2+5x+4}$$
$$-(x^2-3x)$$
$$8x+4$$
$$-(8x-24)$$
$$28 \leftarrow \text{doesn't matter}$$

- no h.a. but a slant asymptote at $\boxed{y=x+8}$
- no holes
- v.a. at $\boxed{x=3}$

$\boxed{\text{x-ints at } x=-4 \text{ and } x=-1}$

9. Polynomial inequality (open and closed intervals)

a) $\frac{(x+2)(x-1)}{(x-3)(x+4)} \geq 0$ ← greater than or equal to

no holes

v.a. at $x=-4, 3$

x-ints at $x=-2, 1$

$(-\infty,-4)$	-4	$(-4,-2)$	-2	$(-2,1)$	1	$(1,3)$	3	$(3,\infty)$
+		−		+		−		+
-5		-3		0		2		4

$\boxed{(-\infty,-4) \cup [-2,1] \cup (3,\infty)}$

(-5) $\frac{(-5+2)(-5-1)}{(-5-3)(-5+4)} \to \frac{(-3)(-6)}{(-8)(-1)} \to +$

(-3) $\frac{(-3+2)(-3-1)}{(-3-3)(-3+4)} \to \frac{(-1)(-4)}{(-6)(1)} \to -$

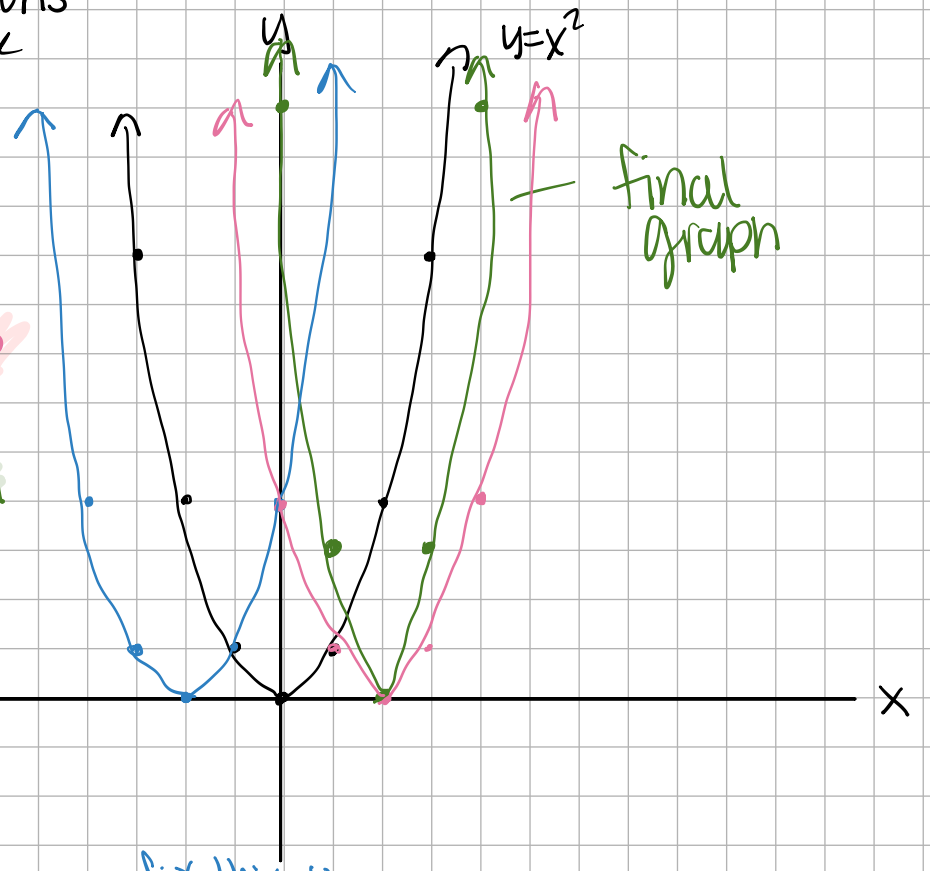
(2) $\frac{(2+2)(2-1)}{(2-3)(2+4)} \to \frac{(4)(1)}{(-1)(6)} \to -$

(0) $\frac{(0+2)(0-1)}{(0-3)(0+4)} \to \frac{(2)(-1)}{(-3)(4)} \to +$

(4) $\frac{(4+2)(4+1)}{(4-3)(4+4)} \to \frac{(6)(3)}{(1)(8)} \to +$

10. Transformations (each square is 1)

parent functions will be in black

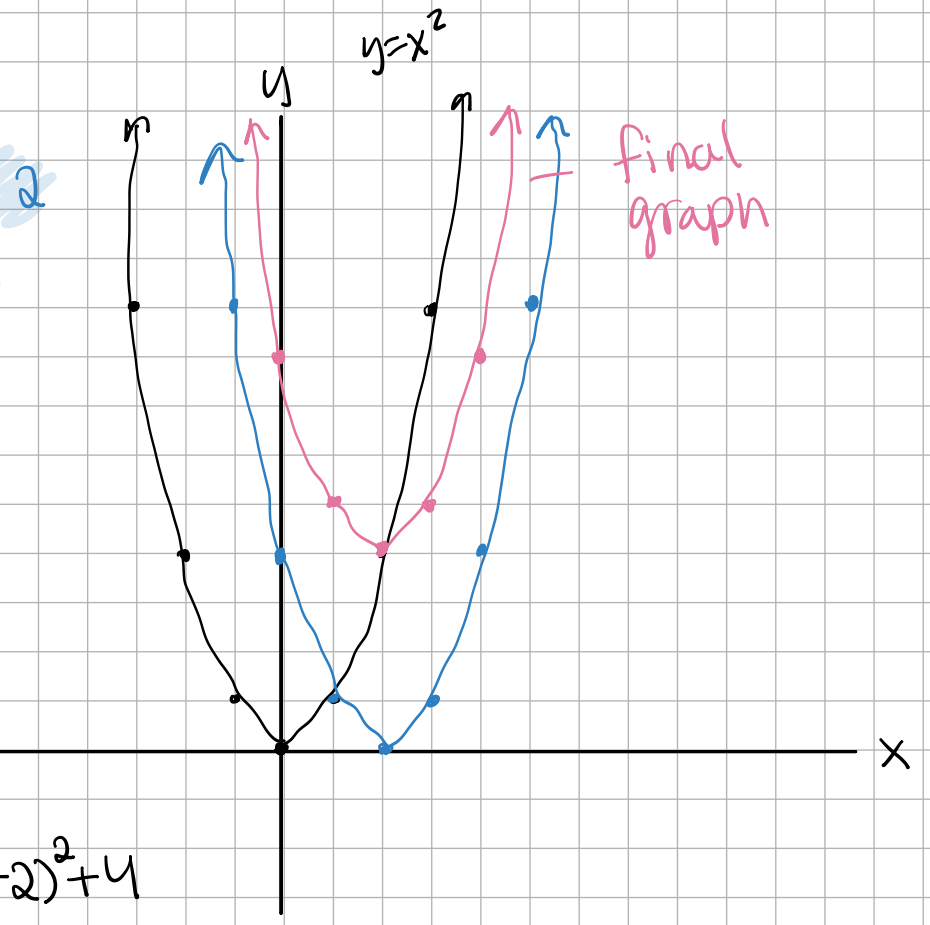


move left 2

reflection across y-axis

vertical stretch of 3

a) $y=3(-x+2)^2$ - fix this ↑



move right 2

move up 4

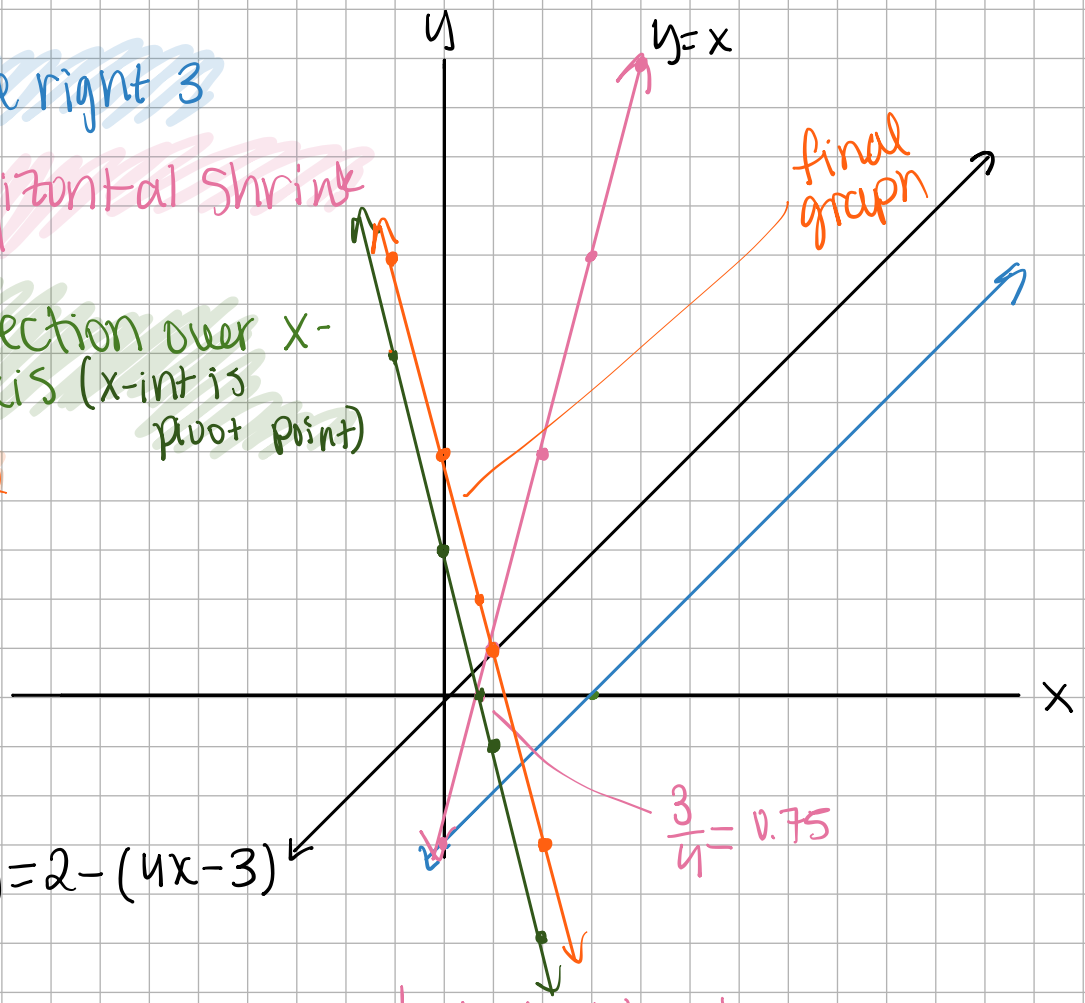
b) $f(x)=(x-2)^2+4$

move right 3

horizontal shrink of 4

reflection over x-axis (x-int is pivot point)

up 2



c) $y = 2 - (4x - 3)$

divide x-values by 4