Name: ______________________ Date: 10/30/25

Test Prep 4

(This test is no calculator, so try your best to do this with no calculator)

1. Proving Identities

a) $\underbrace{\cos^2(\pi/2-y)}\csc y = \sin y$

LHS

$\sin^2 y \cdot \csc y$

$\frac{\sin^{\cancel{2}} y}{1} \cdot \frac{1}{\cancel{\sin y}} \rightarrow \boxed{\sin y} \checkmark$

like tan(-a)

b) $\cot(-a)\cos(-a) + \sin(-a) = -\csc(a)$

LHS

$-\cot(a)\cos(a) + (-\sin(a))$

$-\cot(a)\cos(a) - \sin(a)$

$-\frac{\cos(a)}{\sin(a)} \cdot \frac{\cos(a)}{1}$ $\quad -\frac{\cos^2(a)}{\sin(a)} - \frac{\sin(a)}{1} \cdot \frac{\sin(a)}{\sin(a)}$

$-\frac{\cos^2(a)}{\sin(a)} - \frac{\sin^2(a)}{\sin(a)} \rightarrow \frac{\overbrace{-\cos^2(a) - \sin^2(a)}^{-1}}{\sin(a)} \rightarrow \frac{-1}{\sin(a)} = \boxed{-\csc(a)}$

c) $\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = 1$

$\frac{\frac{\cos x}{1}}{\frac{1}{\cos x}}$ $\quad \frac{\cos x}{1} \cdot \frac{\cos x}{1} \rightarrow \cos^2 x$ $\quad \frac{\frac{\sin x}{1}}{\frac{1}{\sin x}}$ $\quad \frac{\sin x}{1} \cdot \frac{\sin x}{1} \rightarrow \sin^2 x$

$\cos^2 x + \sin^2 x = \boxed{1}$

2. Substitute $\tan\theta$ for x in the expression $\frac{1}{1+x^2}$. Assume $0 \le \theta \le \pi/2$

↑
QI

$$\frac{\sin^2 x+\cos^2 x = 1}{\cos^2 x}$$

$$\underbrace{\tan^2 x+1}_{\text{replace with}} = \sec^2 x$$

$$\frac{1}{1+x^2} \qquad \frac{1}{1+\tan^2 x} \leftarrow$$

$$\frac{1}{\sec^2 x} \rightarrow \boxed{\cos^2 x}$$

3. Find which formula to use, then solve

a) $\overset{s}{\cos 23^\circ} \overset{t}{\cos 67^\circ} - \sin 23^\circ \sin 67^\circ$

$\cos(23^\circ + 67^\circ)$

or

$\boxed{\cos(90^\circ) = 0}$

b) $\frac{\tan 55^\circ - \tan 10^\circ}{1+\tan 55^\circ \tan 10^\circ}$

$\tan(55^\circ - 10^\circ)$

or

$\tan(45^\circ) = 1$

c) $\cos 75^\circ$ $\quad 135-60 \quad 45+30 \quad 225-150$

$\cos(30^\circ+45^\circ) = \cos 30 \cos 45 - \sin 30 \sin 45$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \quad \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

4. Find the exact value of $\sin(s+t)$ given $\sin s = 4/5$, $\underbrace{0<s<\pi/2}_{QI}$, and $\cos t = -6/8$, $\underbrace{\pi/2<t<\pi}_{QII}$

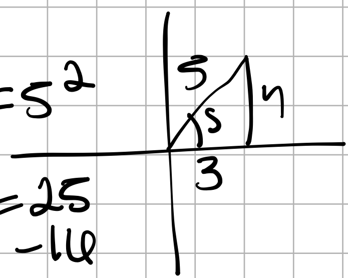
$a^2+4^2=5^2$

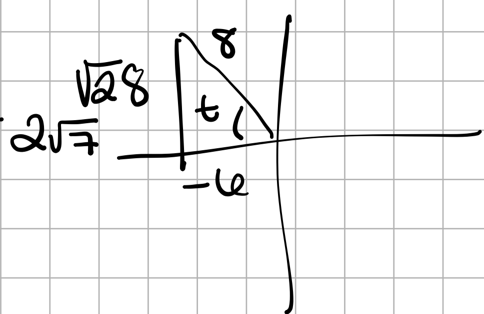
$a^2+16=25$

$-16 \quad -16$

$\sqrt{a^2}=\sqrt{9}$

$a=3$





$(-6)^2+b^2=8^2$

$36+b^2=64$

$-36 \quad -36$

$b^2=28$

$b=\sqrt{28}$

$\sin s \cos t + \cos s \sin t$

$\left(\frac{4}{5}\right)\left(\frac{-6}{8}\right)+\left(\frac{3}{5}\right)\left(\frac{2\sqrt{7}}{8}\right)$

$\frac{-24}{40}+\frac{6\sqrt{7}}{40} \quad \boxed{\frac{-24+6\sqrt{7}}{40}}$

$\sqrt{28}$

14 2

7 2

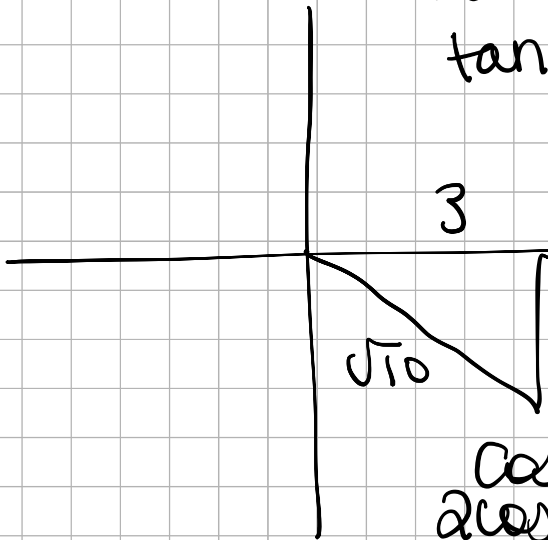
$2\sqrt{7}$

Double Angles :)

5 Find $\sin 2x$, $\cos 2x$ and $\tan 2x$

a) $\tan x = -1/3$, $\cos x > 0$

cos is positive



tan2x

$$\frac{\frac{-3}{5}}{\frac{4}{5}}$$

$\frac{-3}{\cancel{5}} \cdot \frac{\cancel{5}}{4}$ → $\boxed{\tan 2x \quad \frac{-3}{4}}$

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$(-1)^2 + (3)^2 = c^2$

$1 + 9 = c^2$

$10 = c^2$

$\sqrt{10} = c$

cos 2x

$2\cos^2 x - 1$

$2\left(\frac{3}{\sqrt{10}}\right)^2 - 1 \quad 2\left(\frac{9}{10}\right) - 1$

$\frac{18}{10} - \frac{10}{10} = \boxed{\frac{4}{5} \cos 2x}$

simplified

sin 2x

$2\left(\frac{-1}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}\right)$ simplified

$2\left(\frac{-3}{10}\right) \rightarrow \boxed{\frac{-3}{5} \sin 2x}$

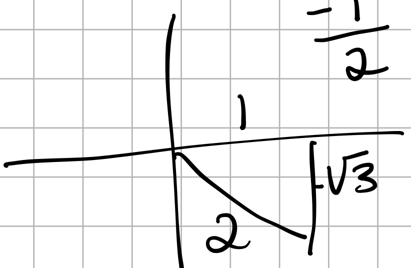
b) $\sec x = 2$, x in Quadrant IV

$\cos x = \frac{1}{2}$

tan2x

$$\frac{\frac{-\sqrt{3}}{2}}{\frac{-1}{2}}$$

$-\frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{-1} = \boxed{\sqrt{3}}$



$1^2 + b^2 = 2^2$

$1 + b^2 = 4$
$-1 \quad\quad -1$

$b^2 = 3$

$b = \sqrt{3}$

sin2x

$2\left(\frac{-\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$1\,\cancel{2}\left(\frac{-\sqrt{3}}{\cancel{2}\,4}\right) \rightarrow \boxed{\sin 2x \quad \frac{-\sqrt{3}}{2}}$

$2\cos^2 - 1$

$2\left(\frac{1}{2}\right)^2 - 1$

$2\left(\frac{1}{4}\right) - 1$

$\frac{2}{4} - 1 \quad \frac{2}{4} - \frac{4}{4}$

$\frac{-2}{4}$ or $\boxed{\frac{-1}{2}}$ cos2x

6. Write $\sin 3x$ in terms of sin

$\sin(2x+x) = \sin 2x \cos x + \cos 2x \sin x$

$2\sin x \cos x(\cos x) + (1-2\sin^2 x)\sin x$

$2\sin x \cos^2 x + \sin x - 2\sin^3 x$

$\cos^2 x + \sin^2 x = 1$
$-\sin^2 x$

$\cos^2 x = 1-\sin^2 x$

$2\sin x(1-\sin^2 x) + \sin - 2\sin^3 x$

$2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$\boxed{-4\sin^3 x + 3\sin x}$

7. Prove the identity

a) $\dfrac{2\tan x}{1+\tan^2 x} = \sin 2x$

LHS

$\dfrac{2\tan x}{1+\tan^2 x}$ $\dfrac{2\tan x}{\sec^2 x}$ $\dfrac{\frac{2\sin x}{\cos x}}{\frac{1}{\cos^2 x}}$

$= \sec^2 x$

$\dfrac{2\sin x}{\cancel{\cos x}} \cdot \dfrac{\cos^2 x}{1}$ $\boxed{2\sin x \cos x = \sin 2x}$

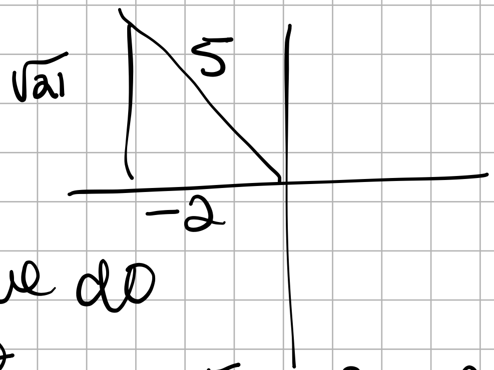
double angle identity

$\overbrace{\phantom{2\cos^{-1}(-2/5)}}^{\sin 2\theta}$

8. Evaluate $\sin\left(2\underbrace{\boxed{\cos^{-1}(-2/5)}}_{}\right)$

just something to point out

let $\cos^{-1}(-2/5) = \theta$



$(-2)^2 + b^2 = 5^2$

$4 + b^2 = 25$

$-4 \qquad -4$

$b^2 = 21$

$b = \sqrt{21}$

now, we do $\sin 2\theta$

$\rightarrow 2\left(\frac{\sqrt{21}}{5}\right)\left(\frac{-2}{5}\right) \quad 2\left(\frac{-2\sqrt{21}}{25}\right) \quad \boxed{\frac{-4\sqrt{21}}{25}}$

Half-angle formulas

9. Find the exact value of $\sin 15°$

$\sin \frac{30°}{2} = \sqrt{\frac{1-\cos 30°}{2}}$

$\sqrt{\frac{1-\sqrt{3}/2}{2}}$

$\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{2}} \quad \sqrt{\frac{\frac{2-\sqrt{3}}{\cancel{2}}}{\cancel{2}}} \Rightarrow \boxed{\frac{1}{2}\sqrt{2-\sqrt{3}}}$

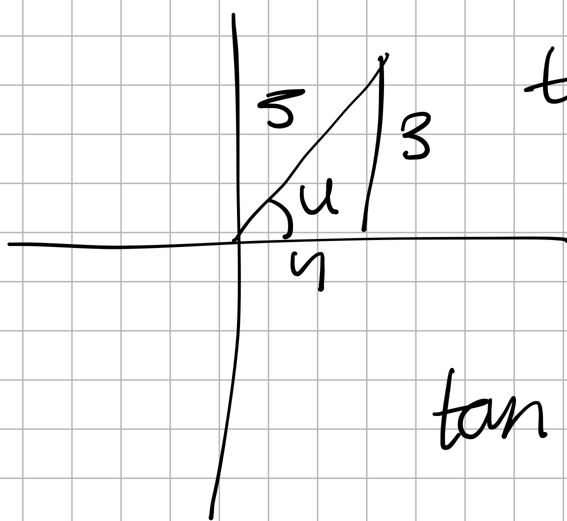
10. Find the exact value of $\cos \underbrace{\pi/12}_{\pi/6}$

$$\cos\frac{\pi/6}{2} = \sqrt{\frac{1+\cos\frac{\pi}{6}}{2}}$$

$$\cos\frac{\pi/6}{2} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} \to \frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2} \to \frac{\frac{2+\sqrt{3}}{2}}{2} \qquad \frac{2+\sqrt{3}}{\cancel{2}}\cdot\frac{\cancel{2}}{1}$$

$$\boxed{\frac{1}{2}\sqrt{2+\sqrt{3}}}$$

11. Given $\sin u = \frac{3}{5}$, find $\tan\left(\frac{u}{2}\right)$



$\tan u = \frac{3}{4}$

$\downarrow$

$$\tan\frac{u}{2} = \frac{\frac{3}{5}}{1+\frac{4}{5}} \qquad \frac{\frac{3}{5}}{\frac{9}{5}}$$

$$\frac{5}{5}+\frac{4}{5}$$

$$\frac{\overset{1}{\cancel{3}}}{\cancel{5}}\cdot\frac{\cancel{5}}{\cancel{9}\,3} = \boxed{\frac{1}{3}}$$

Lowering powers

12. Lower the powers and rewrite the expression in terms of the first power of cosine

a) $\sin^4 x$

$$\sin^4 x = \left(\frac{1-\cos 2x}{2}\right)^2 \quad \frac{1-\cos 2x}{2}\cdot\frac{1-\cos 2x}{2}$$

$$\frac{1-2\cos(2x)+\cos^2(2x)}{4} \leftarrow \quad \cos^2 2x = \frac{1+\cos 2(2x)}{2}$$

$$\frac{1+\cos 4x}{2}$$

$$\frac{1-2\cos(2x)+\left(\frac{1+\cos(4x)}{2}\right)}{4}$$

$$\frac{1-2\cos 2x+\frac{1}{2}+\frac{1}{2}\cos(4x)}{4}$$

$$\frac{\frac{3}{2}-2\cos 2x+\frac{1}{2}\cos 4x}{4}$$

$$\frac{3}{8}-\frac{2}{4}\cos 2x+\frac{1}{8}\cos(4x)$$

$$\boxed{\frac{3}{8}-\frac{1}{2}\cos 2x+\frac{1}{8}\cos(4x)}$$

Product-to-sum formulas

13. Write the products as a sum

a) $\sin 5x \cos 4x$

$$\frac{1}{2}\left[\sin(5x+4x)+\sin(5x-4x)\right]$$

$$\boxed{\frac{1}{2}(\sin 9x+\sin x)}$$

b) $\cos x \sin 4x$

$$\frac{1}{2}\left[\sin(x+4x) - \sin(x-4x)\right]$$

$-3x$

cancel each other

$$\boxed{\frac{1}{2}(\sin 5x + \sin 3x)}$$

Sum-to-product formulas

14. Write the sum as a product

a) $\sin 7x + \sin 5x$

$$2\sin\frac{7x+5x}{2}\cos\frac{7x-5x}{2}$$

$$2\sin\frac{12x}{2}\cos\frac{2x}{2}$$

$$\boxed{2\sin 6x \cos x}$$

b) $\cos 4x - \cos 6x$

$$-2\sin\frac{4x+6x}{2}\sin\frac{4x-6x}{2}$$

$$-2\sin\frac{10x}{2}\sin\frac{-2x}{2}$$

$$-2\sin 5x \sin -x \qquad \boxed{2\sin 5x \sin x}$$

Solving basic trigonometric equations

list solutions from $[0, 2\pi]$

a) $\cos\theta = -\sqrt{3}/2$

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$\boxed{\theta = \frac{5\pi}{6}, 7\pi/6}$

b) $\sin\theta = 1/2$

$\boxed{\theta = \pi/6, 5\pi/6}$

list ALL possible solutions

c) $2\sin^2\theta - \sin\theta - 1 = 0$

let $u = \sin\theta$

$2u^2 - u - 1 = 0$

$u^2 - u - 2 = 0$

$(u - \frac{2}{2})(u - \frac{1}{2}) = 0$

$(u-1)(2u-1) = 0$

$(\sin\theta - 1)(2\sin\theta - 1) = 0$

$\sin\theta - 1 = 0$ $\quad$ $2\sin\theta - 1 = 0$

$\sin\theta = 1$ $\quad$ $+1 \; +1$

$\boxed{\theta = \frac{\pi}{2} + 2k\pi}$ $\quad$ $\frac{2\sin\theta = 1}{2}$

$k \in \mathbb{Z}$ $\quad$ $\sin\theta = 1/2$

$\boxed{\begin{array}{l} \theta = \pi/6 + 2k\pi, \; k \in \mathbb{Z} \\ \theta = \frac{5\pi}{6} + 2k\pi, \; k \in \mathbb{Z} \end{array}}$

d) $\sin^2\theta = 2\sin\theta + 3$

$u^2 = 2u + 3$

$u^2 - 2u - 3$

$(u-3)(u+1)$

$(\sin\theta - 3)(\sin\theta + 1) = 0$

$\sin\theta - 3 = 0$ $\quad$ $\sin\theta + 1 = 0$

$\sin\theta = 3$ $\quad$ $\sin\theta = -1$

not possible

$\boxed{\theta = \frac{3\pi}{2} + 2k\pi, k \in \mathbb{Z}}$