

Worksheet answer key

1. factor the following polynomials

a. $3x^2 + 14x + 8$

"bottoms up" method

$$3x^2 + 14x + 8$$

$$x^2 + 14x + 24$$

$$(x+12)(x+2)$$

$$\frac{3}{3} \quad \leftarrow \quad \frac{1}{3}$$

$$(3x+2)(x+4)$$

$$24 \cdot 1$$

$$12 \cdot 2$$

$$8 \cdot 3$$

$$6 \cdot 4$$

b) $x^2 - 2x - 15$

one has to be positive
one must be negative

$$(x-5)(x+3)$$

$$15 \mid 1$$

$$5 \mid 3$$

c) $5x^2 + 19x + 12$

$$5x^2 + 19x + 12 \quad 5 \cdot 12 = 60$$

$$x^2 + 19x + 60$$

$$60 \cdot 1$$

$$(x+15)(x+4)$$

$$\frac{5}{5} \quad \leftarrow \quad \frac{1}{5}$$

$$30 \cdot 2$$

$$20 \cdot 3$$

$$(x+3)(5x+4)$$

$$15 \cdot 4$$

2. Evaluate the following circle equation, and rewrite the equation in standard form

$$x^2 + y^2 + 6x - 4y - 23 = 0$$

$$(x^2 + 6x + 9) + (y^2 - 4y + 4) = 23 + 9 + 4$$

$6/2 \rightarrow 3^2 = 9$ $-4/2 \rightarrow -2^2 = 4$ $\checkmark$
 $(x+3)^2$ $(y-2)^2$ 36

$$(x+3)^2 + (y-2)^2 = 36 \quad \sqrt{36} = 6 \quad r = 6$$

3. Write the following rational function with the following values

V.o @ $x = 5, 9$

X-int @ $x = -2, -3$

hole @ $x = 4$

$x = -2$
 ~~$x + 2$~~

$x = -3$
 ~~$x + 3$~~

$x = 4$
 -4

$x = 5$
 ~~$x - 5$~~

$x = 9$
 ~~$x - 9$~~

$x - 5 = 0$ - V.o.

$x - 9 = 0$ V.o.

$x + 2 = 0$ x-int

$x + 3 = 0$ x-int

$x - 4 = 0$ hole

$$\frac{(x+2)(x+3)(x-4)}{(x-5)(x-9)(x+4)}$$

4. Evaluate the following rational inequality

$$\frac{3x-1}{x-2} < \frac{5}{x-2}$$

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$$\frac{3x-1-5}{x-2} \Rightarrow \frac{3x-6}{x-2} < 0$$

$$\frac{3x-1}{x-2} - \frac{5}{x-2} < 0$$

Common denominator

Numerator

$$3x-6=0$$

$$\frac{3x-6}{3} \quad x=3$$

less than <

+	2	-	3	+	denominator
$(-\infty, 2)$	$(2, 3)$	$(3, \infty)$			$x-2=0$
1	2.5	4			$x=2$

$$1 \rightarrow \frac{3(1)-6}{1-2} = \frac{3-6}{-1} = \frac{-3}{-1} = 3 \quad \boxed{3}$$

$$2.5 \rightarrow \frac{3(2.5)-6}{2.5-2} = \frac{7.5-6}{0.5} = \frac{1.5}{0.5} = 3 \quad \boxed{-3}$$

$$4 \rightarrow \frac{3(4)-6}{4-2} = \frac{12-6}{2} = \frac{6}{2} = 3 \quad \boxed{3}$$

answer

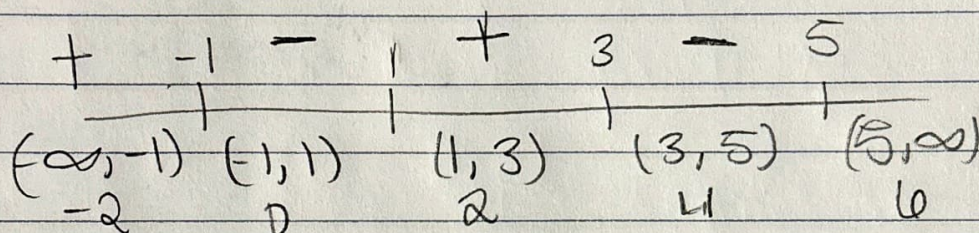
$$\boxed{(2, 3]}$$

5. Evaluate the following rational inequality

$$\frac{x^2 - 4x + 3}{x^2 - 11x - 5} \geq 0$$

$$\frac{(x-3)(x-1)}{(x-6)(x+1)} \geq 0$$

$$\begin{array}{l} x=3 \quad x=1 \quad x \text{ int } 5 \\ x=6 \quad x=-1 \quad \text{v.o.} \end{array}$$



$$\frac{(-2)^2 - 4(-2) + 3}{(-2)^2 - 4(-2) - 5} \rightarrow 2.1429 \quad +$$

$$\frac{0^2 - 4(0) + 3}{0^2 - 4(0) - 5} \rightarrow -0.6 \quad - \quad \boxed{(-\infty, -2) \cup [2, 4] \cup (6, \infty)}$$

$$\frac{(2)^2 - 4(2) + 3}{(2)^2 - 4(2) - 5} \rightarrow 0.1111 \quad +$$

$$\frac{4^2 - 4(4) + 3}{4^2 - 4(4) - 5} \rightarrow -0.6 \quad -$$

$$\frac{(6)^2 - 4(6) + 3}{6^2 - 4(6) - 5} \rightarrow 2.1429 \quad +$$

6. analyze the following rational functions:

$$\frac{x^2 + 8x + 12}{x + 2} \quad \frac{(x+6)(x+2)}{x+2}$$

$$\begin{array}{r} x+6 \\ x+2 \overline{) x^2 + 8x + 12} \\ \underline{-(x^2 + 2x)} \\ 6x + 12 \\ \underline{-(6x + 12)} \\ 0 \end{array}$$

Slant asymptote
 $y = x + 6$
 hole at $x = -2$
 x-int at $x = -6$

7. Use polynomial long division to divide $x^3 + 9x^2 + 23x + 15$ by $(x+1)$. Use the answer to find the rest of the factors

$$\begin{array}{r} x^2 + 8x + 15 \\ x+1 \overline{) x^3 + 9x^2 + 23x + 15} \\ \underline{-(x^3 + x^2)} \\ 8x^2 + 23x \\ \underline{-(8x^2 + 8x)} \\ 15x + 15 \\ \underline{-(15x + 15)} \\ 0 \end{array}$$

$$\boxed{x^2 + 8x + 15 = (x+3)(x+5)(x+1)}$$