Name: Answer key

Date: 10/27/25

Session 11

Proving identities

a) $\frac{\tan\theta}{\sin\theta} = \sec\theta$

LHS

$\frac{\tan\theta}{\sin\theta}$ $\qquad \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{1}}$ $\qquad \frac{\cancel{\sin\theta}}{\cos\theta} \cdot \frac{1}{\cancel{\sin\theta}}$

✓

$\frac{1}{\cos\theta}$ or $\boxed{\sec\theta}$

b) $\cos(-x) - \sin(-x) = \cos x + \sin x$

LHS

$\cos(-x) - \sin(-x)$

$\cos(x) - (-\sin(x))$

or ✓

$\boxed{\cos x + \sin x}$

Evaluate each expression under the given conditions:

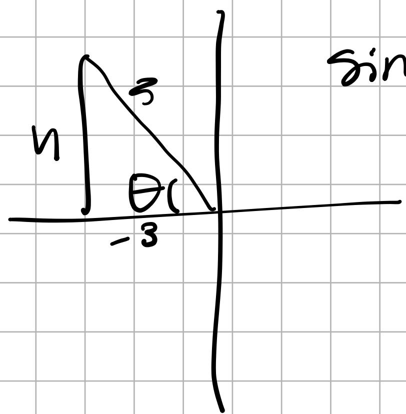
a) $\sin(\theta - \phi)$; $\sin\theta = 4/5$ in QII
$\tan\phi = 1/3$ in QIII

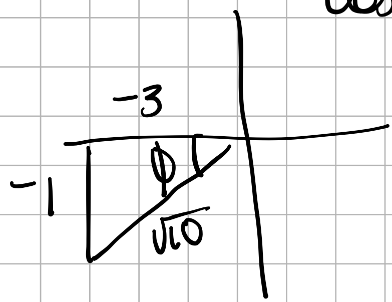
$\sin\phi = \frac{-1}{\sqrt{10}}$ or $\frac{-\sqrt{10}}{10}$

$\cos\theta = \frac{-3}{5}$

$\sin\theta = \frac{4}{5}$

$\cos\phi = \frac{-3}{\sqrt{10}}$ or $\frac{-3\sqrt{10}}{10}$





$1^2 + 3^2 = c^2$

$1 + 9 = c^2$

$\sqrt{10} = \sqrt{c^2}$

$\sqrt{10} = c$

$\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$

$$\left(\frac{4}{5}\right)\left(\frac{-3\sqrt{10}}{10}\right) - \left(\frac{-3}{5}\right)\left(\frac{-\sqrt{10}}{10}\right)$$

$$\frac{-12\sqrt{10}}{50} - \left(\frac{3\sqrt{10}}{50}\right)$$

$$\frac{-12\sqrt{10}}{50} - \frac{3\sqrt{10}}{50} \qquad \frac{\overset{-3}{\cancel{-15}}\sqrt{10}}{\underset{10}{\cancel{50}}} \qquad \boxed{\frac{-3\sqrt{10}}{10}}$$

Find sin2x, cos2x and tan2x if

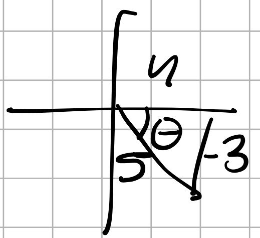
a) $\cos x = 4/5$, $\csc < 0$

$\uparrow$

$\sin < 0$

S | ~~A~~

T | C $\leftarrow$ cos is positive



$\sin 2x$

$2\left(\frac{-3}{5}\right)\left(\frac{4}{5}\right)$

$2\left(\frac{-12}{25}\right)$

$2\cos^2 x - 1$

$2(4/5)^2 - 1$

$2(16/25) - 1$

$32/25 - \frac{25}{25}$ $\rightarrow$ $7/25$

$\sin 2x = \boxed{\frac{-24}{25}}$

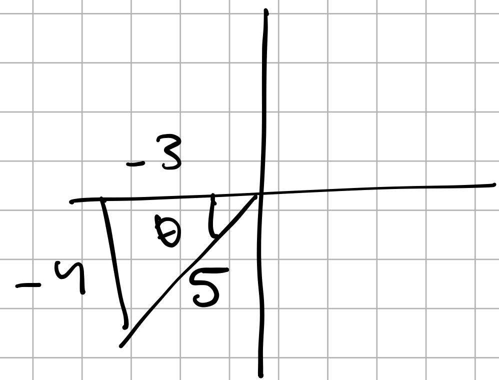
$\cos 2x = \boxed{\frac{7}{25}}$

$\tan 2x = \boxed{\frac{-24}{7}}$

$\frac{2\left(\frac{-3}{4}\right)}{1-(-3/4)^2}$ $\quad \frac{\frac{-6}{4}}{1 - 9/16}$ $\quad \frac{16}{16} - \frac{9}{16} = \frac{7}{16}$ $\quad \frac{-6/4}{\frac{7}{16}}$

$\frac{-6}{1\cancel{4}} \cdot \frac{\cancel{16}^{4}}{7}$ $\boxed{\frac{-24}{7}}$

b) $\sin x = -3/5$, x in Quadrant III



$2(-4/5)(-3/5)$

$2(12/25)$

$\boxed{\begin{array}{l} \sin 2x = 24/25 \\ \cos 2x = -7/25 \\ \tan 2x = -24/7 \end{array}}$

$2(-3/5)^2 - 1$

$2(9/25) - 1$

$\frac{18}{25} - \frac{25}{25} \rightarrow \frac{-7}{25}$

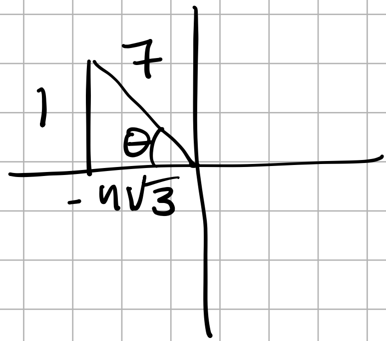
$\frac{2(4/3)}{1-(4/3)^2}$ $\quad \frac{\frac{8}{3}}{\frac{9}{9} - \frac{16}{9}} \rightarrow \frac{-7}{9}$

$\frac{8}{1\cancel{3}} \cdot \frac{\cancel{9}^{3}}{-7}$

$\frac{24}{-7}$

$\frac{8/3}{1-16/9}$

c) $\sin x = 1/7$ in QII $\leftarrow$ only find $\sin 2\theta$



$1^2 + b^2 = 7^2$

$1 + b^2 = 49$

$-1 \quad -1$

$\sqrt{b^2} = \sqrt{48}$

$b = 4\sqrt{3}$

or $-4\sqrt{3}$

$48 < 8 < 4 < 2,\ 2 \quad 6 < 3,\ 2 \quad 4 < 2,\ 2$

$2 \cdot 2\sqrt{3}$

$2(1/7)\left(\frac{-4\sqrt{3}}{7}\right)$

$$\boxed{-\frac{8\sqrt{3}}{49}}$$

If we have time :)

Prove the identity (Using half-angle formulas)

a) $\dfrac{\sin x + \sin y}{\cos x + \cos y} = \tan\left(\dfrac{x+y}{2}\right)$

LHS

Sum to product rules

$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

$$\frac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}$$

$$\frac{2\cdot\sin\frac{x+y}{2}\cancel{\cos\frac{x-y}{2}}}{2\cos\frac{x+y}{2}\cancel{\cos\frac{x-y}{2}}}$$

$$\frac{\cancel{2}\sin\frac{x+y}{2}}{\cancel{2}\cos\frac{x+y}{2}} \qquad \frac{\sin\frac{x+y}{2}}{\cos\frac{x+y}{2}} \to \boxed{\tan\left(\frac{x+y}{2}\right)}$$