Name: Answer key Date: 11/17/25

Session 15

Dot product reminders!!!

If you have $\vec{a} = \langle a_1, a_2 \rangle$ and $\vec{b} = \langle b_1, b_2 \rangle$,

then your formula is

$$\vec{a} \cdot \vec{b} = (a_1 b_1) + (a_2 b_2)$$

your answer should just be a number, <u>not a vector!</u>

If you have an angle between $\vec{a}$ and $\vec{b}$, then

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta \quad \text{or} \quad \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

If $\vec{a}$ and $\vec{b}$ are perpendicular (orthogonal), then and <u>only then</u>,

$$\vec{a} \cdot \vec{b} = 0$$

↑ regular zero

properties of dot product

$$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u} \qquad (c\vec{u})\vec{v} = c(\vec{u} \cdot \vec{v}) = \vec{u}(c\vec{v})$$

$$(\vec{u} + \vec{v})\vec{w} = \vec{u} \cdot \vec{w} + \vec{v} \cdot \vec{w} \qquad |\vec{u}|^2 = \vec{u} \cdot \vec{u}$$

1. Determine the Dot product of the two vectors and the angle between them!

$\vec{a} = 3i + 4j$ ↓ $\vec{a} = \langle 3, 4 \rangle$

$\vec{b} = 5i - 2j$ ↓ $\vec{b} = \langle 5, -2 \rangle$

$\vec{a} \cdot \vec{b} = (3 \cdot 5) + (4 \cdot -2)$

$15 + (-8)$

7

$|\vec{a}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$|\vec{b}| = \sqrt{5^2 + (-2)^2} = \sqrt{29} = \sqrt{29}$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} \rightarrow \cos\theta = \dfrac{7}{5 \cdot \sqrt{29}} \rightarrow \cos\theta = 0.2599734$

$\cos^{-1}(0.2599734) = \theta$

$\theta = 74.93°$ ← make sure your calculator is in deg. mode!

2. A forklift moves 34 m, carrying a 1032 N box across the floor. How much work is being done by the forklift?

$W = \vec{F} \cdot \vec{d}$

$\vec{F} = \langle 1032, 0 \rangle$

$\vec{d} = \langle 34, 0 \rangle$

We have no force or distance working in the y-direction

$\vec{F} \cdot \vec{d} = (1032 \cdot 34) + (0 \cdot 0)$

↓

35,088 J

J is N·m but are used to lb-ft Still force · distance, just different quantities

partial fraction decomposition

3. Write the form of the partial fraction decomposition. Do NOT determine the numerical value of the coefficients

a) $\frac{1}{(x-1)(x+2)}$

$$\frac{A}{x-1} + \frac{B}{x+2}$$

b) $\frac{x^2-3x+5}{(x-2)^2(x+4)}$

$$\frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+4}$$

c) $\frac{x^3-4x^2+2}{(x^2+1)(x^2+2)}$

$$\frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2}$$

4. Write the form of the partial fraction decomposition. Now, determine the values of the coefficients

a) $\frac{2}{(x-1)(x+1)}$

$\frac{1}{x-1} - \frac{1}{x+1}$

$A=1$

$\frac{(x-1)(x+1)}{1} \cdot \frac{2}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} \cdot \frac{(x-1)(x+1)}{1}$

$2 = A(x+1) + B(x-1)$

$x=1$ $x=-1$

plugging in A we just found

$2 = A(1+1) + B(1-1)$

$2 = 1(-1+1) + B(-1-1)$

$\frac{2}{2} = \frac{A(2)}{2}$ $A=1$

$\frac{2}{-2} = \frac{B(-2)}{-2}$ $B=-1$

b) $\frac{x+14}{x^2-2x-8}$

factor first

$\frac{3}{x-4} - \frac{2}{x+2}$

$(x-4)(x+2)$

(if you're ever unsure, foil your guess)

$\frac{(x-4)(x+2)}{1} \cdot \frac{x+14}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2} \cdot \frac{(x-4)(x+2)}{1}$

$x+14 = A(x+2) + B(x-4)$ lets try $x=4$

$4+14 = A(4+2) + B(4-4)$

$\frac{18}{6} = \frac{A(6)}{6}$ $A=3$

now lets try $x=-2$

$-2+14 = 3(-2+2) + B(-2-4)$

$\frac{12}{-6} = \frac{B(-6)}{-6}$ $B=-2$

Extra paper