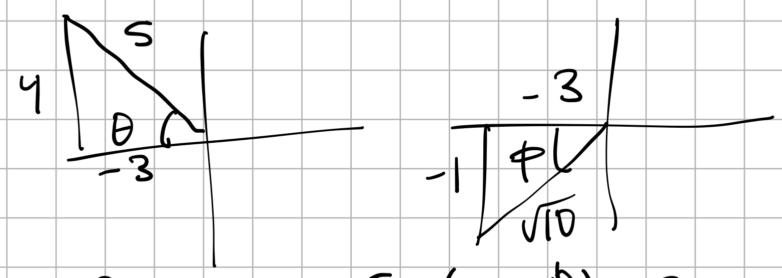
Name: Answer key Date: 3/30/26

Session 11

1. Find $\sin(\theta - \phi)$, given $\sin\theta = 4/5$ in QII and $\tan\phi = 1/3$ in QIII



$(-3)^2 + (-1)^2 = c^2$

$9 + 1 = c^2$

$\oplus\ \sqrt{10} = c$

$5^2 = 4^2 + b^2$

$\pm 3 = b$

$\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$

$\left(\frac{4}{5}\right)\left(\frac{-3}{\sqrt{10}}\right) - \left(\frac{-3}{5}\right)\left(\frac{-1}{\sqrt{10}}\right)$

$\frac{-12}{5\sqrt{10}} - \left(\frac{3}{5\sqrt{10}}\right) \quad \frac{-12}{5\sqrt{10}} - \frac{3}{5\sqrt{10}} \rightarrow \frac{-15}{5\sqrt{10}} \rightarrow \boxed{-\frac{3}{\sqrt{10}}}$

2. Prove the following expression

$$\frac{\sin x + \sin y}{\cos x + \cos y} = \tan\left(\frac{x+y}{2}\right)$$

$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

$$\frac{\cancel{2}\sin\frac{x+y}{2}\cancel{\cos\frac{x-y}{2}}}{\cancel{2}\cos\frac{x+y}{2}\cancel{\cos\frac{x-y}{2}}}$$

$$\frac{\sin\frac{x+y}{2}}{\cos\frac{x+y}{2}} \rightarrow \boxed{\tan\frac{x+y}{2}}$$

3. Write the products as a sum

$\sin 5x \cos 4x$

$\frac{1}{2}[\sin(u+v) + \sin(u-v)]$

$\underset{9x}{5x+4x} \quad \underset{x}{5x-4x}$

$\boxed{\frac{1}{2}[\sin 9x + \sin x]}$

4. Write the sum as a product

$\sin 7x + \sin 5x$

$2\sin\frac{7x+5x}{2}\cos\frac{7x-5x}{2}$ $\quad \frac{12x}{2} \quad \frac{2x}{2}$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$\boxed{2\sin 6x \cos x}$

5. prove the following identity

$\frac{\cos(2x)-1}{\sin(2x)} = -\tan x$

$$\cos 2x = 1 - 2\sin^2 x$$

$\frac{\cancel{1}-2\sin^2 x\cancel{-1}}{\underbrace{\sin(2x)}_{=2\sin x\cos x}} \quad \frac{-2\sin^2 x}{2\sin x\cos x} \rightarrow \frac{-\cancel{2(\sin x)}(\sin x)}{\cancel{2\sin x}\cos x} = \frac{-\sin x}{\cos x}$

$\boxed{= -\tan x}$

Lowering powers

6. Lower the powers and rewrite the expression in terms of the first power of cosine

$\sin^4 x$

$$\sin^4 x = \left(\frac{1-\cos(2x)}{2}\right)^2 = \frac{1-2\cos(2x)+\cos^2(2x)}{4}$$

$$\frac{1}{4}-\frac{1}{2}\cos(2x)+\frac{1}{4}\cos^2(2x) \qquad \cos^2 x = \frac{1+\cos 2x}{2}$$

$$\cos^2(2x) = \frac{1+\cos(4x)}{2}$$

$$\frac{1}{4}-\frac{1}{2}\cos(2x)+\frac{1}{4}\left(\frac{1+\cos(4x)}{2}\right)$$

$$\underline{\frac{1}{4}}-\frac{1}{2}\cos(2x)+\underline{\frac{1}{8}}+\frac{1}{8}\cos(4x)$$

$$\frac{2}{8}+\frac{1}{8}$$

$$\boxed{\frac{3}{8}-\frac{1}{2}\cos(2x)+\frac{1}{8}\cos(4x)}$$