Name: Answer Key Date: 10/20/25

Session 10 ☺

Warm up: Write all of your trig expressions (you can also write any other helpful info)

I'll start: $\sin^2\theta+\cos^2\theta=1$

$\tan^2\theta+1=\sec^2\theta$

$1+\cot^2\theta=\csc^2\theta$

$\sin(-x)=-\sin(x)$ ← odd

$\cos(-x)=\cos(x)$ ← even

$\tan(-x)=-\tan(x)$ ← odd

$\frac{1}{\sin\theta}=\csc\theta$

$\frac{1}{\cos\theta}=\sec\theta$

$\cot\theta=\frac{\cos\theta}{\sin\theta}$

$\tan\theta=\frac{\sin\theta}{\cos\theta}$

$\sin(\pi/2-x)=\cos x$

$\cos(\pi/2-x)=\sin x$

$\tan(\pi/2-x)=\cot x$

$\cot(\pi/2-x)=\tan x$

$\sec(\pi/2-x)=\csc x$

$\csc(\pi/2-x)=\sec x$

1. prove that $\cot(x)+\tan(x)=\sec(x)\csc(x)$

LHS: $\cot(x)+\tan(x)$

$\frac{\cos}{\cos}\cdot\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}\frac{\sin}{\sin}$

$\frac{\cos^2 x}{\cos x\sin x}+\frac{\sin^2 x}{\cos x\sin x}\rightarrow\frac{\cos^2 x+\sin^2 x}{\cos x\sin x}$ ($\cos^2x+\sin^2x=1$)

$\frac{1}{\cos x\sin x}\rightarrow\frac{1}{\cos x}\cdot\frac{1}{\sin x}$

$\sec x\csc x$

2. prove that $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\csc$

LHS

$\frac{\sin\theta}{\sin\theta}\cdot\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}$

$(1+\cos)(1+\cos)$

$1+2\cos\theta+\cos^2\theta$

$\frac{\sin^2\theta}{(\sin\theta)(1+\cos\theta)}+\frac{(1+\cos\theta)^2}{(\sin\theta)(1+\cos\theta)}$

$\frac{\sin^2+(1+\cos\theta)^2}{(\sin\theta)(1+\cos\theta)}$

$\frac{\sin^2\theta+1+2\cos\theta+\cos^2\theta}{(\sin\theta)(1+\cos\theta)}$ ($\sin^2\theta+\cos^2\theta=1$)

$\frac{1+1+2\cos\theta}{(\sin\theta)(1+\cos\theta)}$ ($2+2\cos\theta$)

$\frac{2(1+\cos\theta)}{(\sin\theta)(1+\cos\theta)}$

$\frac{2}{\sin\theta}$ or $2\csc$

3. Prove that $\cos x + \sin x \tan x = \sec x$

LHS

$$\frac{\cos x}{1} + \frac{\sin x}{1} \cdot \frac{\sin x}{\cos x}$$

$\cos^2 x$

$$\frac{\cos x \cos x + \sin^2 x}{\cos x} \rightarrow \frac{\overbrace{\cos^2 x + \sin^2 x}^{=1}}{\cos x} \rightarrow \frac{1}{\cos x} \text{ or } \sec x$$

Now we're going to write our additive and subtractive property formulas here ↓

Sine:

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\sin(u-v) = \sin u \cos v - \cos u \sin v$

Cosine:

$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$\cos(u-v) = \cos u \cos v + \sin u \sin v$

tangent:

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

4. Find the exact value of $\sin 75$
(having a unit circle is very helpful)

$$\sin(30+45) = \sin 30\cos 45 + \cos 30 \sin 45$$
$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$\boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$

5. Find $\tan 105$ (there are many right processes. Just make sure your answer matches in the calculator!!

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(60+45) = \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1} \qquad \boxed{\frac{\sqrt{3}+1}{1-\sqrt{3}}}$$ exact form