

Test prep 1 answer key

1. Foil the following factors

$$(2x+5)(x-9) \text{ FOIL}$$

combine like terms

$$2x^2 - 18x + 5x - 45$$

$$\boxed{2x^2 - 13x - 45}$$

2. Factor the following polynomials

a) $4x^2 + 13x - 12$

b) $x^2 - x - 30$

$$4x^2 + 13x - 12$$

$$x^2 + 13x - 48$$

$$(x+16)(x-3)$$

48	1
24	2
16	3
12	4
8	6

$$(x-6)(x+5)$$

30	1
15	2
10	3
6	5

$$\boxed{(x+1)(4x-3)}$$

3. Evaluate the following circle equation, and rewrite the equation into standard form.

$$x^2 + y^2 - 8x - 6y - 24 = 0$$

$$(x^2 - 8x + 16) + (y^2 - 6y + 9) = 24 + 16 + 9$$

$$-8/2 \Rightarrow -4^2 = 16, \quad -6/2 \Rightarrow -3^2 = 9$$

$$\boxed{(x-4)^2 + (y-3)^2 = 49}$$

$$\sqrt{r^2} = \sqrt{49}$$

$$r = 7$$

4. Find an equation of the circle that has the points $P(1, 8)$ and $Q(5, -6)$ as the endpoints of a diameter.

Find the midpoint of points to find center of circle

$$\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \quad h, k \quad \frac{1+5}{2}, \frac{8-6}{2} \rightarrow \frac{6}{2}, \frac{2}{2}$$

$$6/2 \Rightarrow 3 \quad 2/2 \Rightarrow 1 \quad \underline{3, 1} \text{ center}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-3)^2 + (y-1)^2 = r^2$$

$$(1-3)^2 + (8-1)^2 = r^2$$

$$(x-3)^2 + (y-1)^2 = 53$$

$$(-2)^2 + (7)^2 = r^2$$

$$4 + 49 = r^2$$

$$r = \sqrt{53}$$

$$53 = r^2$$

5. The point $(1, -2)$ lies on a circle whose center is at $(-3, -5)$. Write the standard form of this circle equation, given these points.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(1+3)^2 + (-2+5)^2 = r^2$$

$$4^2 + 3^2 = r^2$$

$$\sqrt{25} = r$$

$$5 = r$$

$$(x-(-3)) + (y-(-5))$$

$$(x+3)^2 + (y+5)^2 = r^2$$

$$(x+3)^2 + (y+5)^2 = 25$$

6. Find the difference quotient of $f(x) = x^2 - 4x + 7$

$$\frac{(x+h)^2 - 4(x+h) + 7 - (x^2 - 4x + 7)}{h}$$

cancel these

$$\frac{(x+h)(x+h) - 4x - 4h + 7 - x^2 + 4x - 7}{h}$$

$$\frac{x^2 + 2xh + h^2 - 4x - 4h + 7 - x^2 + 4x - 7}{h}$$

$$\frac{2xh + h^2 - 4h}{h} = \frac{h(2x + h - 4)}{h}$$

$$2x + h - 4$$

7. For the function $f(x) = (x-5)^2$ whose graph is shown below, find the net change and average rate of change between the following values of x .

a) $x=1$ and $x=4$

$$(1-5)^2$$

$$(4-5)^2$$

$$(-4)^2$$

$$(-1)^2$$

$$y=16$$

$$y=1$$

$$\text{Net Change} = y_2 - y_1$$

$$y_2 = 1 \quad y_1 = 16$$

$$1 - 16 = -15$$

$$\frac{1 - 16}{4 - 1}$$

$$\frac{-15}{3}$$

$$-5$$

net change: -15

average rate of change: -5

$$\frac{y_2 - y_1}{x_2 - x_1}$$

8. Evaluate the following rational functions

a)

$$\frac{2x^2 - 8x - 24}{x^2 - 5x - 6}$$

$$2(x^2 - 4x - 12)$$

$$\frac{2(x-6)(x+2)}{(x+1)(x-6)}$$

$$\begin{array}{r} 48 \overline{) 148} \\ 24 \overline{) 148} \\ 12 \overline{) 148} \\ 12 \overline{) 148} \\ 28 \end{array}$$

hole at $x=6$

located at

$(6, 14/7)$

$$\frac{2(6+2)}{6+1}$$

$$\frac{2(8)}{7}$$

$$14/7$$

Vertical asymptote

@ $x=-1$

X-int at $x=-2$

Horizontal asymptote at $y=2$

b) $\frac{x^2 + 5x + 4}{x-3}$

$$\frac{(x+4)(x+1)}{x-3}$$

$$\begin{array}{r} x+8 \\ x-3 \overline{) x^2 + 5x + 4} \\ -(x^2 + 3x) \\ \hline 8x + 4 \\ -(8x + 24) \\ \hline -20 \end{array}$$

no holes

V. a. = $x=3$

X-ints @ $x=-1, -4$

no h. a. Slant asymptote

@ $y=x+8$

$$c) \frac{4x-8}{x^2-3x-4} \quad \frac{4(x-2)}{(x-4)(x+1)}$$

$$h.o.a. @ y=0$$

$$\cancel{4=0}$$

$$x-2=0$$

$$v.o.a. @ x=4, -1$$

$$x=2 - x \text{ int}$$

$$x \text{ ints} @ x=2$$

$$x-4=0$$

$$x=4 - v.o.a.$$

$$x+1=0$$

$$x=-1 - v.o.a.$$

$$a. \frac{(x+2)(x-1)}{(x-3)(x+4)} > 0$$

$$+$$

$$-$$

$$+$$

$$\begin{array}{ccccccc} (-\infty, -4) & (-4, -2) & (-2, 1) & (1, 3) & (3, \infty) \\ -5 & -3 & 0 & 2 & 4 \end{array}$$

$$x=3, -4 \text{ v.o.a.}$$

$$x=-2, 1 \text{ x ints}$$

$$(-\infty, -4) \cup [-2, 1] \cup (3, \infty)$$

v.o.a. x-int
parenthesis brackets

10. Use polynomial long division to divide $x^3+9x^2+23x+15$ by $(x+1)$. Use answers to find remaining factors.

$$\begin{array}{r} x^2+8x+15 \\ x+1 \overline{) x^3+9x^2+23x+15} \\ \underline{-(x^3+x^2)} \\ 8x^2+23x \\ \underline{-(8x^2+8x)} \\ 15x+15 \\ \underline{-(15x+15)} \\ 0 \end{array}$$

$$\begin{array}{r} x^2+8x+15 \\ (x+3)(x+5) \end{array} \quad \begin{array}{r} 15 \overline{) 15} \\ 5 \overline{) 3} \end{array}$$

$$(x+3)(x+5)(x+1)$$

11. Evaluate the following logarithms

a) $\frac{1}{n} \cdot \log_2 81$

$\log_3 \sqrt[n]{81}$ $\sqrt[n]{81} = 3$
 $\log_3 3$
 $\textcircled{1}$

b) $\ln(3x+1)$
 $\log_e(3x+1)$

$\frac{\log_{10} 3x+1}{\log_{10} e}$

c) $\log_2 8^{33}$

$33 \cdot \log_2 8 \Rightarrow 33 \cdot 3 \Rightarrow \boxed{99}$

12. Evaluate the following exponential expressions

a) $\frac{5 \cdot 4^{3-2x}}{5} = 8$

$4^{3-2x} = 8/5$

$\log_n 8/5 = 3-2x$
 $-3 \quad -3$

$-3 + \log_n 8/5$ or $(\log_n 8/5) - 3$

$(\log_n 8/5) - 3 = -2x$

$\frac{(\log_n 8/5) - 3}{-2} = x$

$$\begin{aligned} & \triangleright b) 10^{5x} = 24 \\ & \triangleright \log_{10} 24 = 5x \\ & \triangleright \frac{\log_{10} 24}{5} = x \end{aligned}$$

$$\begin{aligned} & \triangleright c) 3^{x+5} = 4 \\ & \triangleright \log_3 4 = x+5 \\ & \triangleright (\log_3 4) - 5 = x \end{aligned}$$

$$\begin{aligned} & \bullet \triangleright 12. \text{ Solve } e^{2x} - e^x - 6 = 0 \\ & \triangleright \text{Substitute } e^x \text{ for } u \\ & \triangleright u^2 - u - 6 = 0 \\ & \triangleright (u-3)(u+2) = 0 \\ & \triangleright u = 3 \quad u = -2 \\ & \triangleright \boxed{e^x = 3} \quad \boxed{e^x = -2} \\ & \triangleright \boxed{\ln 3 = x} \quad \boxed{\ln -2} \end{aligned}$$

$$\begin{aligned} & \triangleright 13. \text{ Solve } \log_5(x^2+1) = \log_5(x-2) + \log_5(x+3) \\ & \triangleright \text{drop } \log_5 \\ & \triangleright x^2+1 = (x-2)(x+3) \\ & \triangleright x^2+1 = x^2+x-6 \\ & \triangleright x^2+1 \quad x^2 \quad +6 \\ & \triangleright \boxed{7 = x} \end{aligned}$$

$$14. A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$7\% = 0.07 = r$$

$$n = \text{"annually"} = 1$$

$$P = 4,583.00$$

$$t = 9 \text{ (years)}$$

$$4,583.00 \left(1 + \frac{0.07}{1}\right)^{1 \cdot 9}$$

$$\boxed{\$8,425.66}$$

$$15. A = Pe^{rt}$$

$$r = 6.5\% = 0.065$$

$$t = 36 \text{ months} = 3 \text{ years}$$

$$A = \$7,500$$

$$A =$$

$$7500 = Pe^{0.065 \cdot 3}$$

$$7500 = P e^{0.195}$$

$$7500 = P e^{0.065 \cdot 3}$$

$$7500 = P (1.2153 \approx)$$

$$1.2153 \approx$$

$$\$6171.26 = P$$