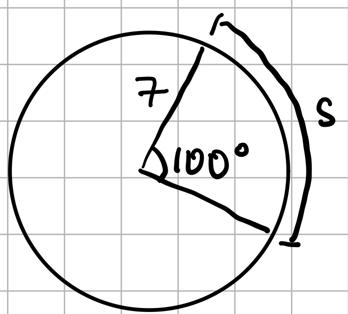
Name: Answer key Date: 10/14/25

Unit 3 Test Prep

☆ Warmup: Solve for the missing variable

a)

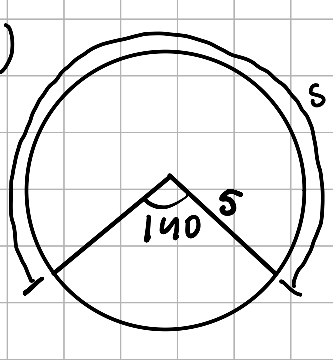


$\frac{100°}{1} \cdot \frac{\pi}{180°}$

$s = r\theta$ — must be in radians

$\boxed{\frac{5\pi}{9} \cdot 7 = 12.22}$

b)



$\frac{140°}{1} \cdot \frac{\pi}{180°}$

$\frac{7\pi}{9}$

$C = 2\pi r$

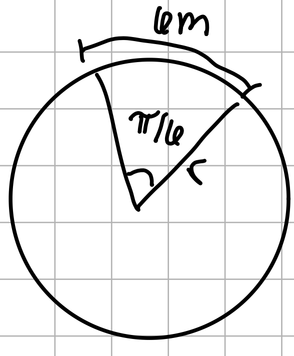
$C = 2\pi(5)$

$C = 31.415$

$s = 5 \cdot \frac{7\pi}{9} = 12.22$

$\boxed{31.415 - 12.22 = 19.23}$

c)

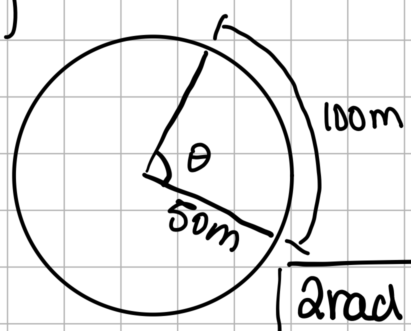


$s = r\theta$

$\frac{6m = r(\pi/6)}{\pi/6}$

$\boxed{r = 11.45m}$

d)



$\boxed{2\text{rad or } 114.59°}$

(θ in radians and degrees)

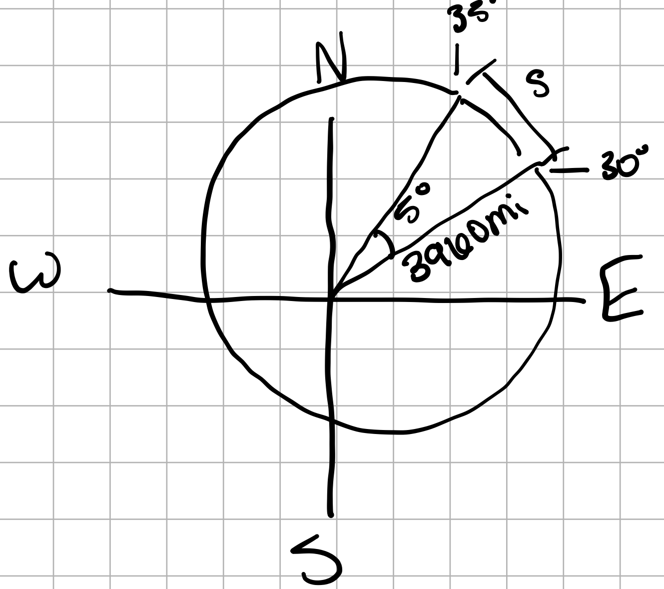
$s = r\theta$

$\frac{100m = 50m\,\theta}{50m}$

$2\text{rad} = \theta$

$\frac{2\text{rad}}{1} \cdot \frac{180°}{\pi\,\text{rad}}$

1. Memphis, TN and New Orleans, LA lie approximately on the same meridian (longitude line). Memphis has a latitude 35°N and New Orleans has a latitude of 30°N. Find the distance between these cities if the radius of the Earth is 3960 mi.



$35-30=5°$

$\frac{5°}{1}\cdot\frac{\pi}{180°}=\frac{\pi}{36}$

$s=r\theta$

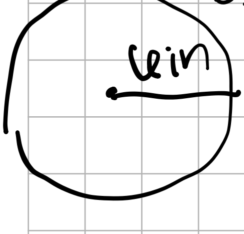
$s=3960\times\frac{\pi}{36}=\boxed{110\pi \text{ mi or } 345.58 \text{ mi}}$

2. A radial saw has a blade with a 6-inch radius. Suppose the blade spins at 1000 rpm

a) Find the angular speed of the blade in rad/min

$$\frac{1000 \text{ rev}}{1 \text{ min}}\cdot\frac{2\pi}{1}=\boxed{\frac{2000\pi \text{ rad}}{\text{min}}}$$

b) Find the linear velocity of the saw in ft/sec



$$\frac{12\pi \text{ in}}{1}\cdot\frac{1000 \text{ rev}}{1 \text{ min}}\rightarrow\frac{12{,}000\pi \text{ in}}{\text{min}}\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)$$

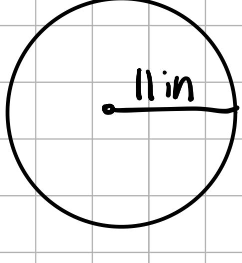
$\boxed{52.36 \text{ ft/sec}}$

$V=\frac{s}{t}$ s - distance, t - time

$2\pi\times 6\text{in}=12\pi \text{ in}$

distance travelled in one rev

3. The wheels of a car have a diameter of 22 inch and are rotating at 600 rpm. Find the speed of the car in mi/hr



$2r = d$

$\frac{2r = 22in}{2}$

$r = 11in$

$v = \frac{s}{t}$

make sure you put the π in your calc

$\frac{22\pi in}{1} \cdot \frac{600 rev}{1 min} = \frac{13200\pi \cancel{in}}{\cancel{min}} \left(\frac{60 \cancel{min}}{1 hr}\right)\left(\frac{1 \cancel{ft}}{12 \cancel{in}}\right)\left(\frac{1 mi}{5280 \cancel{ft}}\right)$

$2\pi \times 11in = 22\pi in$

distance travelled

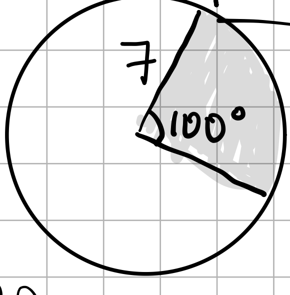
only one rev, but you want 600 rev, so multiply by 600

$\boxed{V = 39.27 mi/hr}$

4. Find the area of the shaded region

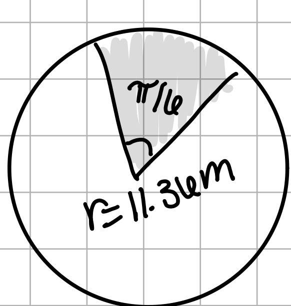
a) $\frac{1}{2}(7)^2 \cdot \frac{5\pi}{9}$

$\boxed{A = 42.76}$



b) $\frac{1}{2}(11.36m)^2 \frac{\pi}{6}$

$\boxed{33.79 m^2}$



$A = \frac{1}{2} r^2 \theta$ must be in radians

c)

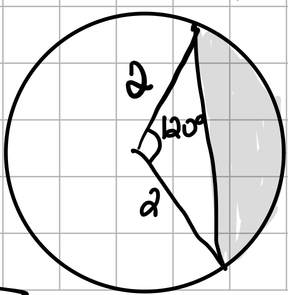
area of sector

$A = \frac{1}{2}(2)^2\left(\frac{2\pi}{3}\right)$

$A = 4.19$

$A = \frac{1}{2}(2)(2) \sin 100$

1.97

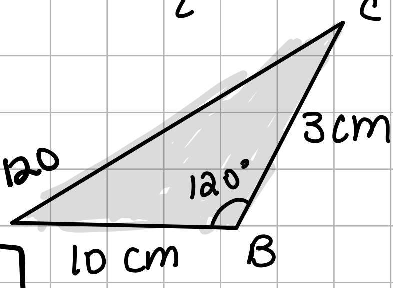


$4.19 - 1.97 = 2$

$\boxed{2.22}$

use $A = \frac{1}{2} ab \sin \theta$

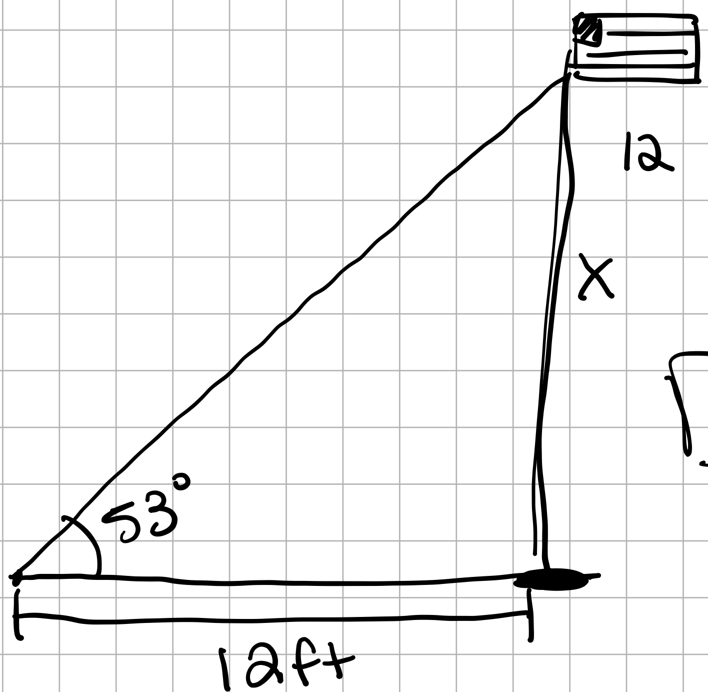
d)



$\frac{1}{2}(3cm)(10cm) \sin 120$

$\boxed{A = 12.99 cm^2}$

5. From a point on the ground 12 ft from the base of a flagpole, the angle of elevation of the top of the pole measures 53°. How tall is the flagpole?



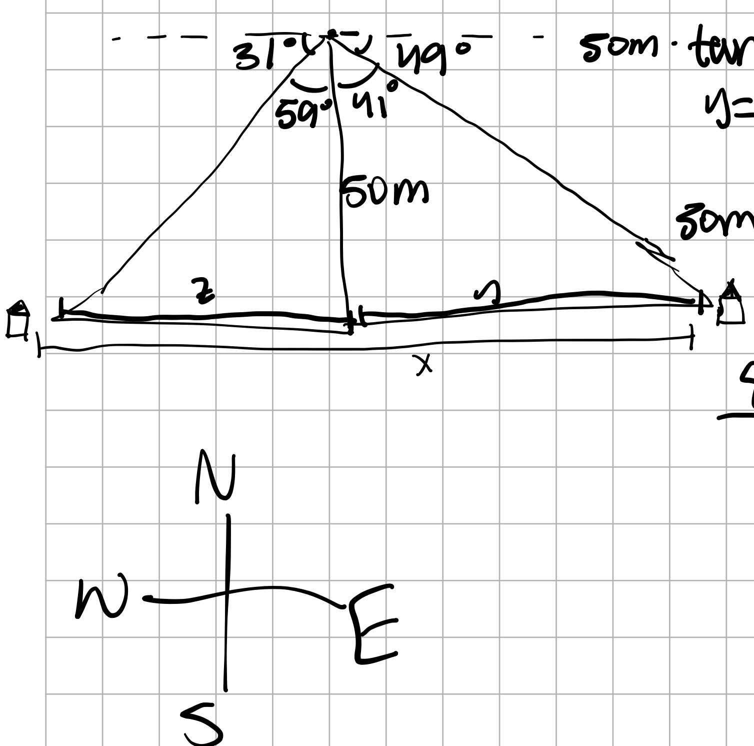
$12 \cdot \tan 53° = \frac{x}{12} \cdot 12$

$12 \cdot \tan 53° = x$

$\boxed{x = 15.92\text{ft}}$

6. An observer at the top of a 50 m tower notes that the angle of depression of a house lying due east is 49° and the angle of depression of a house lying due west is 31°. Find the distance between the houses.

$x = y + z$ $\boxed{x = 126.68\text{m}}$



$50\text{m} \cdot \tan 41 = \frac{y}{50\text{m}} \cdot 50\text{m}$

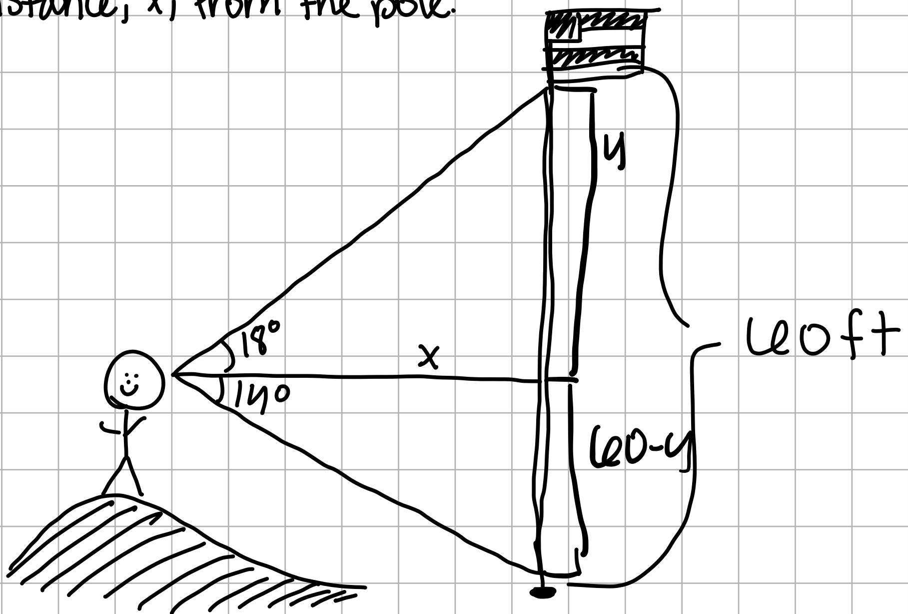
$y = 43.46\text{m}$

$50\text{m} \cdot \tan 59° = \frac{z}{50} \cdot 50\text{m}$

$z = 83.21\text{m}$

$83.21\text{m} + 43.46\text{m} = 126.68\text{m}$

7. A hiker standing on a hill sees a 60ft tall flagpole. The angle of depression to the bottom of the pole is 14°, and the angle of elevation to the top of the pole is 18°. Find the hiker's distance, x, from the pole.



We have to do a system of equations

$$x \cdot \tan 18° = \frac{y}{x} \cdot x \qquad x \cdot \tan 14° = \frac{60-y}{x} \cdot x$$

$x \cdot \tan 18° = y$

↑ plug this in

$x \cdot \tan 14° = 60 - y$

$x \cdot \tan 14° = 60 - x \cdot \cancel{\tan 18°}$

$+ x \cdot \tan 18 \qquad + \cancel{x} \tan 18$

$x \tan 14° + x \tan 18 = 60$

$$\frac{x(\tan 14 + \tan 18) = 60}{\tan 14 + \tan 18}$$

→ $\boxed{x = 104.5\text{ft}}$

Trig Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

another good bit of information to remember:

SOHCAHTOA

8. Express $\sin\theta$ in terms of $\cos\theta$

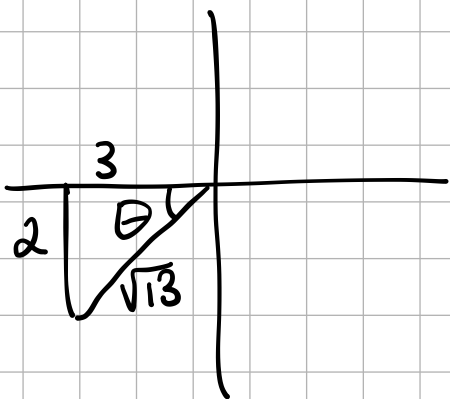
$$\sin^2\theta+\cos^2\theta=1$$
$$-\cos^2\theta \qquad -\cos^2\theta$$
$$\sqrt{\sin^2\theta}=\sqrt{1-\cos^2\theta}$$
$$\boxed{\sin\theta=\sqrt{1-\cos^2\theta}}$$

tan > 0

9. Given that $\cot\theta=\frac{3}{2}$ and $\cos\theta<0$, find the other five trig functions.

S | A
T | C

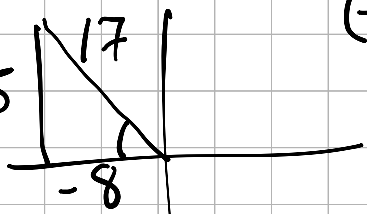
Q III



$$\tan\theta=\boxed{\frac{2}{3}}$$
$$\sin\theta=\frac{2}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}}=\boxed{\frac{2\sqrt{13}}{13}}$$
$$\cos\theta=\frac{3}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}}\rightarrow\boxed{\frac{3\sqrt{13}}{13}}$$
$$\csc\theta=\boxed{\frac{\sqrt{13}}{2}}$$
$$\sec\theta=\boxed{\frac{\sqrt{13}}{3}}$$

let $\cos^{-1}(-8/17)=\theta$

10. Find $\sin(\cos^{-1}(-8/17))$



$$(-8)^2+b^2=17^2$$
$$64+b^2=289$$
$$-64 \qquad -64$$
$$\sqrt{b^2}=\sqrt{225}$$
$$b=15$$ positive

$$\boxed{\sin\theta=\frac{15}{17}}$$

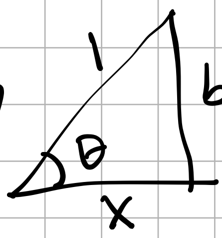
11. Write $\sin(\cos^{-1}(x))$ as an algebraic expression in x

$\sin(\underbrace{\cos^{-1}(x)})$

let $\cos^{-1}(x) = \theta$

$\cos\theta = x \rightarrow \frac{x}{1} \quad \frac{adj}{hyp}$

$$\boxed{\sin\theta = \frac{\sqrt{1-x^2}}{1} \text{ or } \sqrt{1-x^2}}$$



$(x)^2 + b^2 = 1^2$

$x^2 + b^2 = 1$

$-x^2 \qquad -x^2$

$\sqrt{b^2} = \sqrt{1-x^2}$

$b = \sqrt{1-x^2}$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

you'll be given one variation of each formula

Law of Cosines

$a^2 = b^2 + c^2 - 2bc \sin A$

$b^2 = c^2 + a^2 - 2ca \sin B$

$c^2 = a^2 + b^2 - 2ab \sin C$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

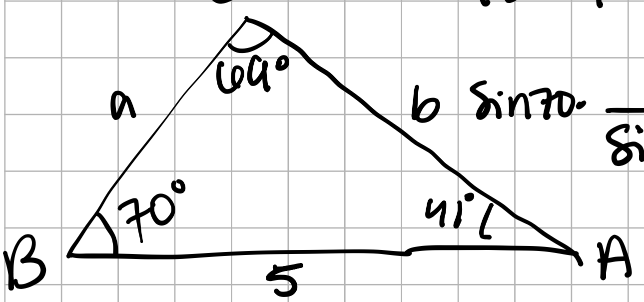
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

12. All cases of Law of Sines. Solve the triangles

a) ASA → $\measuredangle A = 41°$, $\measuredangle B = 70°$, and $c = 5$

$180 - 70 - 41 = 69°$

$$\boxed{\begin{array}{c} \measuredangle C = 69° \\ b = 5.03 \\ a = 3.51 \end{array}}$$



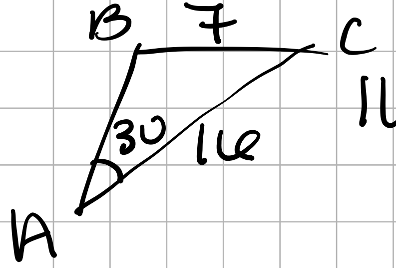
$$\sin 70 \cdot \frac{5}{\sin 69} = \frac{b}{\cancel{\sin 70}} \cdot \cancel{\sin 70}$$

$b = 5.03$

$$\sin 41 \cdot \frac{5}{\sin 69} = \frac{a}{\cancel{\sin 41}} \cdot \cancel{\sin 41}$$

$a = 3.51$

b) SSA: No solution → $a = 7$, $\measuredangle A = 30°$, $b = 16$



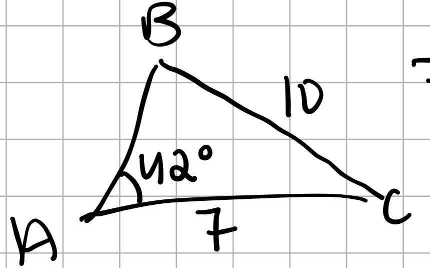
$$16 \cdot \frac{\sin 30}{7} = \frac{\sin B}{16} \cdot 16$$

$\sin B = 1.1428$

$\sin^{-1}(1.1428) = \boxed{\text{No solution}}$

Will show up on calc as "Error: Domain"

c) SSA: One solution → $a = 10$, $\measuredangle A = 42°$, $b = 7$



$$7 \cdot \frac{\sin 42}{10} = \frac{\sin B}{7} \cdot 7$$

$\sin B = 0.46839$

$\sin^{-1}(0.46839) = B$

$\measuredangle B = 27.929°$

$$\boxed{\begin{array}{c} \measuredangle B = 27.93° \\ \measuredangle C = 110.07° \\ c = 14.04 \end{array}}$$

$180 - 27.93 - 42 = \underline{110.07}$

$180 - 27.93 = 152.07°$

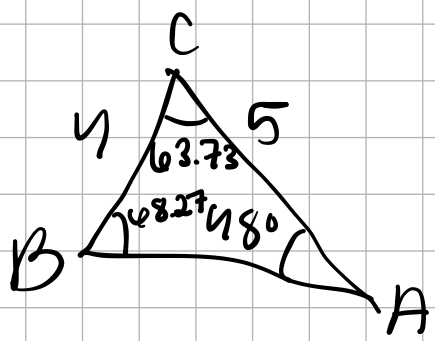
$$\sin 110.07 \cdot \frac{10}{\sin 42} = \frac{c}{\cancel{\sin 110.07}} \cdot \cancel{\sin 110.07}$$

$180° - 152.07° - 42° = \underline{-14.07°}$

$c = 14.04$

$\boxed{\text{not possible only one triangle}}$

a) SSA: two solutions → $a=4$, $\measuredangle A = 48°$, $b=5$



$$5 \cdot \frac{\sin 48}{4} = \frac{\sin B}{\cancel{5}} \cdot \cancel{5}$$

$$\sin 63.73 \cdot \frac{c}{\cancel{\sin 63.73}} = \frac{4}{\sin 48} \cdot \sin 63.73$$

$\sin B = 0.92893$

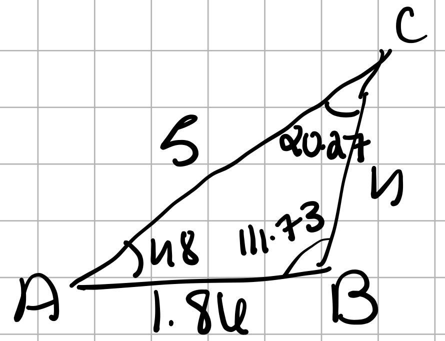
$\sin^{-1}(0.92893) = B$ $\quad c = 4.83$

$180 - 68.27 = \underset{\measuredangle B}{111.73°}$

$\boxed{\begin{array}{l} \measuredangle B = 68.27 \\ \measuredangle C = 63.73 \\ c = 4.83 \end{array}}$

$180 - 111.73 - 48 = \underset{\measuredangle C}{20.27}$

$$\frac{c}{\sin 20.27} = \frac{4}{\sin 48}$$

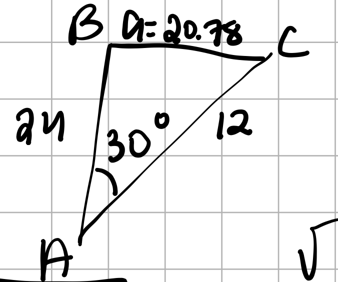


triangle 2

$\boxed{\begin{array}{ll} \measuredangle A = 48 & \measuredangle C = 20.27 \\ \measuredangle B = 111.73 & c = 1.86 \end{array}}$

13. All cases for law of cosines. Solve the triangles

a) SAS → $\measuredangle A = 30°$, $b=12$, $c=24$



$a^2 = b^2 + c^2 - 2bc \sin A$

$a^2 = (12)^2 + (24)^2 - 2(12)(24)\sin 30$

$\sqrt{a^2} = \sqrt{432}$

$a = 20.78$

$180 - 30° - 30° = 120°$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{(20.78)^2 + (24)^2 - (12)^2}{2(20.78)(24)}$$

$\cos B = (0.8660)$

$\cos^{-1}(0.8660) = B$

$\measuredangle B = 30°$

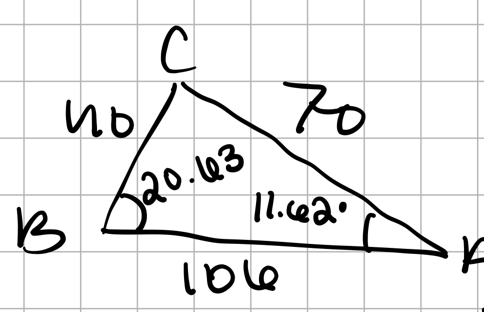
$\boxed{\begin{array}{l} a = 20.78 \\ \measuredangle B = 30° \\ \measuredangle C = 120° \end{array}}$

b) SSS → $a=40$, $b=70$, $c=106$

$$\cos A = \frac{70^2 + 106^2 - 40^2}{2(70)(106)}$$

$180 - 20.63 - 11.62$

$\boxed{\measuredangle C = 147.75}$



$\cos A = (0.9795)$

$\cos^{-1}(0.9795) = A$

$\boxed{\measuredangle A = 11.62°}$

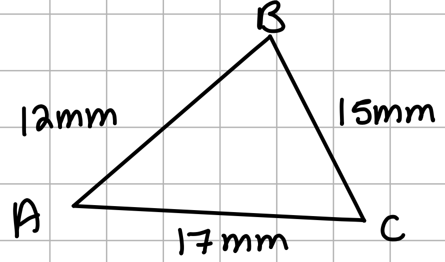
$\cos^{-1}(0.93585) = B$

$\boxed{\measuredangle B = 20.63}$

$$\cos B = \frac{106^2 + 40^2 - 70^2}{2(106)(40)} \to 0.93585$$

14. Find the area of the triangles

a)



$s = \frac{12+17+15}{2} = 22$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{22(22-12)(22-15)(22-17)}$

$\boxed{A = 87.75 mm^2}$

b) $a=4, b=7, c=9$

$s = \frac{4+7+9}{2} = 10$

$A = \sqrt{10(10-4)(10-7)(10-9)}$

$\boxed{A = 13.42}$