Name: Answer Key Date: 11/13/25

Session 14

Review of parametric equations!

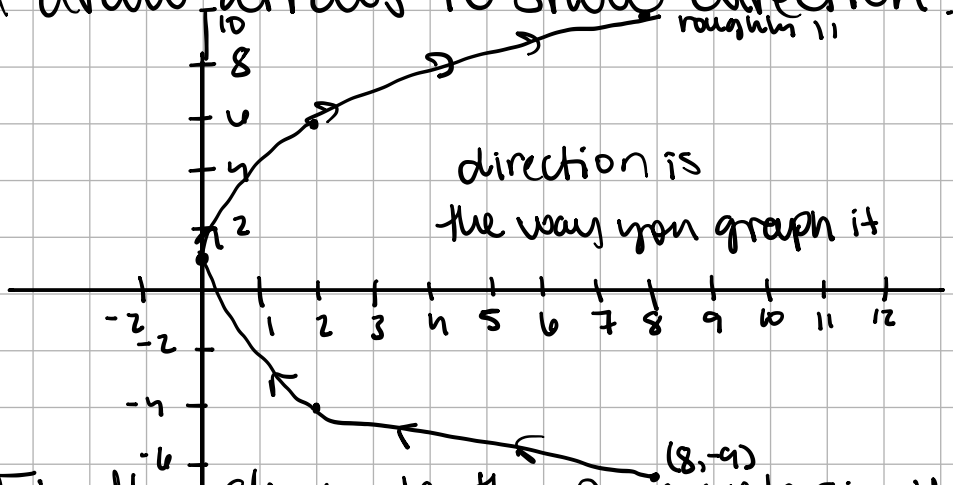
Create a table for the following parametric equations! $-2 \le t \le 2$

$x = 2t^2$

$y = 5t + 1$

t	x	y
-2	8	-9
-1	2	-4
0	0	1
1	2	6
2	8	11

Now, graph the function. Determine its direction and draw arrows to show direction!



Finally, eliminate the parameter in the following parametric equations!

(it doesn't matter if it's $x =$ or $y =$, whatever is easier for you to solve for!)

$x = 2t^2$ I'm going to solve for y

$y = 5t + 1$

$y = 5t + 1$

$-1 \quad -1$

$y - 1 = 5t$

$\frac{y-1}{5} = t$

$x = 2\left(\frac{y-1}{5}\right)^2$

$\frac{(y-1)^2}{5^2} \quad y^2 - 2y + 1$

$x = 2\left(\frac{y^2 - 2y + 1}{25}\right) \rightarrow \boxed{x = \frac{2y^2 - 4y + 2}{25}}$

Now, we move on to vectors!!

What is the definition of a vector?

a quantity that has both direction and magnitude!

Useful properties to know!
(as you practice, these will become less scary, I promise!!)

$\vec{u}+\vec{v}=\vec{v}+\vec{u}$

$\vec{u}+(\vec{v}+\vec{w})=(\vec{u}+\vec{v})+\vec{w}$

zero vector $\vec{0}=\langle 0,0\rangle$

$\vec{u}+\vec{0}=\vec{u}$

$\vec{u}+(-\vec{u})=\vec{0}$

$|c\vec{u}|=c|\vec{u}|$

$c(\vec{u}+\vec{v})+c\vec{u}+c\vec{v}$

$(c+d)\vec{u}=c\vec{u}+d\vec{u}$

$(cd)\vec{u}=c(d\vec{u})=d(c\vec{u})$

$1\cdot\vec{u}=\vec{u}$

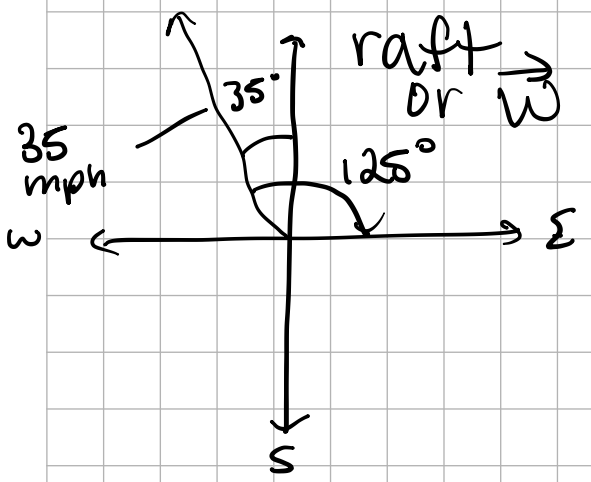
$0\cdot\vec{u}=\vec{0}$

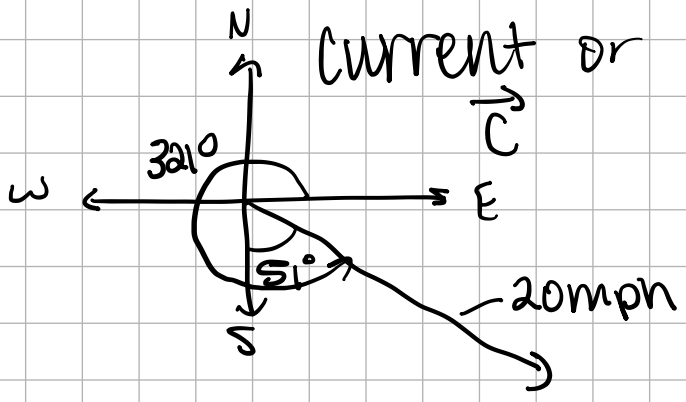
$c\cdot\vec{0}=\vec{0}$

Some more helpful stuff to know about vectors!!

$|\vec{a}|=\sqrt{(a_1)^2+(a_2)^2}$

The white raft you are in is travelling at a speed of 35mph heading N 35°W. The current of the water is 20mph heading S51°E. What is the resultant vector and the direction?





$\vec{w} = (|\vec{w}| \cdot \cos\theta, |\vec{w}| \cdot \sin\theta)$ (x, y)

$\vec{w} = (35 \cdot \cos 125, 35 \cdot \sin 125)$

$\vec{w} = \langle -20.08, 28.67 \rangle$

$\vec{C} = (|\vec{C}| \cdot \cos\theta, |\vec{C}| \cdot \sin\theta)$

$(20 \cdot \cos 321, 20 \cdot \sin 321)$

$\vec{C} = \langle 15.54, -12.59 \rangle$

-312.01

resultant vector

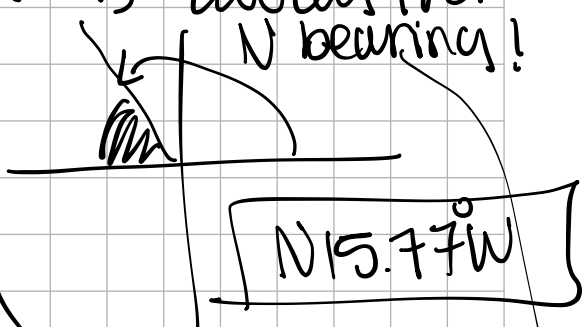
$\vec{w} + \vec{C}$

$\vec{r} = (-20.08 + 15.54), (28.67 + (-12.59)$

$\vec{r} = \langle -4.54, 16.08 \rangle$

component form

15.77 is the amount its away from N bearing!

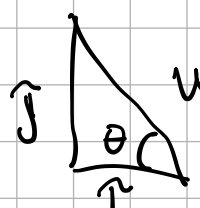


N15.77°W

we also want magnitude

$\sqrt{(-4.54)^2 + 16.08^2}$

16.71mph



$\theta = \tan^{-1}\left(\frac{16.08}{-4.54}\right) = -74.23$

$+90 = 15.77$

Dot product!

If you have $\vec{a} = \langle a_1, a_2 \rangle$ and $\vec{b} = \langle b_1, b_2 \rangle$,

then your formula is

$$\vec{a} \cdot \vec{b} = (a_1 b_1) + (a_2 b_2)$$

your answer should just be a number, <u>not a vector!</u>

If you have an angle between $\vec{a}$ and $\vec{b}$, then

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta \quad \text{or} \quad \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

If $\vec{a}$ and $\vec{b}$ are perpendicular (orthogonal), then and <u>only then</u>,

$$\vec{a} \cdot \vec{b} = 0$$

↑ regular zero

Properties of dot product

$$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u} \qquad (c\vec{u})\vec{v} = c(\vec{u} \cdot \vec{v}) = \vec{u}(c\vec{v})$$

$$(\vec{u} + \vec{v})\vec{w} = \vec{u} \cdot \vec{w} + \vec{v} \cdot \vec{w} \qquad |\vec{u}|^2 = \vec{u} \cdot \vec{u}$$

Determine the Dot product of the two vectors and the angle between them!

$\vec{a} = 3i + 4j$ $\qquad$ $\vec{b} = 5i - 2j$

$\vec{a} = \langle 3, 4 \rangle$ $\qquad$ $\vec{b} = \langle 5, -2 \rangle$

$\vec{a} \cdot \vec{b} = (3 \cdot 5) + (4 \cdot -2)$

$15 + (-8)$

7

$|\vec{a}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$|\vec{b}| = \sqrt{5^2 + (-2)^2} = \sqrt{29} = \sqrt{29}$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} \rightarrow \cos\theta = \dfrac{7}{5 \cdot \sqrt{29}} \rightarrow \cos\theta = 0.25919734$

$\cos^{-1}(0.25919734) = \theta$

$\theta = 74.93°$ ← make sure your calculator is in deg. mode!

Determine the Dot product of the two vectors and the angle between them!

$\vec{a} = \langle 15, 6, -10 \rangle$ $\qquad$ $\vec{b} = \langle 2, 5, -3 \rangle$

$\vec{a} \cdot \vec{b} = (15 \cdot 2) + (6 \cdot 5) + (-10 \cdot -3)$

$30 + 30 + 30$

$\vec{a} \cdot \vec{b} = 90$

$|\vec{a}| = \sqrt{15^2 + 6^2 + (-10)^2} = 19$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$|\vec{b}| = \sqrt{2^2 + 5^2 + (-3)^2}$

$\sqrt{38}$

$\cos\theta = \dfrac{90}{19 \cdot \sqrt{38}} \rightarrow \cos\theta = 0.768417$

$\cos^{-1}(0.768417) = \theta$

$\theta = 39.79$