Name: Answer key Date: 11/24/25

Test prep 5

1. Find a formula for the following arithmetic sequence

3, 7, 11, 15, 19, 23, ...

+4 +4 +4 +4 +4

$d=4$

$a_1=3$

$a_n=3+(n-1)4$

$a_n=3+4n-4$

$\boxed{a_n=4n-1}$

2. Find the nth term of the sequence, then find the 20th term

a) $a_1=2$ and $d=3$

$a_n=2+(n-1)3$

$a_n=2+3n-3$

$a_n=3n-1$

$a_{20}=3(20)-1$

$a_{20}=60-1$ $\boxed{59}$

3. Find the 150^{th} partial sum of the arithmetic sequence $\underline{\quad n \quad}$

$5, 16, 27, 38, 49, \ldots$

$11 \quad 11 \quad 11$

$n = 150$
$a = 5$
$d = 11$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{150}{2}[2(5) + (150-1)11]$

$S_n = 75[10 + (149)11]$

$S_n = 75(10 + 1639)$

$S_n = 75(1649) \rightarrow \boxed{123,675}$

(i don't expect you all to see big numbers like this on the test, but knowing the set-up is important)

4. Find a_1 and r for the following geometric sequence

$3, 6, 12, 24, 48, \ldots$

$r = \frac{a_2}{a_1} \qquad r = \frac{6}{3} = 2$

$a_1 = 3 \quad r = 2$

$\boxed{a_n = 3(2)^{n-1}}$

5. Find the partial sum S_6 for the geometric sequence with $a_1 = 2$ and $r = 3$

$S_n = a\left(\frac{1-r^n}{1-r}\right)$ $\quad n = 6$

$S_n = 2\left(\frac{1-3^6}{1-3}\right)$

$S_n = \cancel{2}\left(\frac{1-729}{\cancel{-2}}\right)$

$S_n = \frac{-728}{-1}$ $\quad \boxed{S_n = 728}$

6. Find the sum of the following infinite geometric sequence

$14, 7, 7/2, 7/4, \ldots$

$S = \frac{a}{1-r}$ $\quad r < 1$ or this doesn't work

$r = \frac{a_2}{a_1} = \frac{7}{14} = \frac{1}{2}$ not on your formula sheet!

$a = 14$

KCF

$S = \frac{14}{1-1/2} \rightarrow \frac{14}{1/2}$ $\quad \frac{14}{1} \cdot \frac{2}{1} \rightarrow \boxed{28}$

using a table

7. Parametric equations: graph the following parametric equations, then eliminate the parameter to find one equation

$x = t + 1$ $\qquad y = \frac{t}{t+1}$

t	x	y
0	1	0
1	2	1/2
2	3	2/3
3	4	3/4
4	5	4/5

$x = t + 1$
$-1 \quad -1$
$t = x - 1$

$y = \frac{x-1}{(x-1)+1}$

$y = \frac{x-1}{x}$

8. Find the parametric equations, given a slope of (1/2) passing through (4, -1)

$\frac{1}{2}$ $\frac{\text{rise}}{\text{run}}$ $\frac{y}{x}$

$x = 4 + 2t$

$y = -1 + t$

9. Find the following values below
(be sure to pay attention to your values and their vector/scalar quantity)

$\vec{a} = 3i + 4j$ $\quad\quad$ $\vec{b} = 5i - 2j$

a) $2(\vec{a} \cdot \vec{b})$

$\vec{a} \cdot \vec{b} = (3 \cdot 4) + (5 \cdot -2)$

$12 - 10$

$2(2)$

$\boxed{4}$

b) $3\vec{a} + \vec{b}$

$3\langle 3, 4 \rangle$

$3\vec{a} = \langle 9, 12 \rangle$

$(9+5, 12+-2)$

$3\vec{a} + \vec{b} = \boxed{\langle 14, 10 \rangle}$

I moved up 4

10. A forklift moves 10ft, carrying a 30lbs box across the floor. How much work is being done by the forklift?

$W = \vec{F} \cdot \vec{D}$

$|\vec{F}| = 30\text{lbs}$

only moving in x-direction so

$|\vec{D}| = 10\text{ft}$

$\vec{F}_x = 30 \cdot \cos 0$

$\vec{F}_y = 30 \cdot \sin 0$

$\vec{F} = \langle 30, 0 \rangle$

$\vec{D}_x = 10 \cdot \cos 0$

$\vec{D}_y = 10 \cdot \sin 0$

$\vec{D} = \langle 10, 0 \rangle$

$\vec{F} \cdot \vec{D} = \langle 30 \cdot 10 + 0 \cdot 0 \rangle$

$\boxed{W = 300\text{lb-ft}}$

11. Write the form of the partial fraction decomposition. Do not determine the numerical value of the coefficients

a) $\frac{x^2-3x+5}{(x-2)^2(x+4)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+4}$

b) $\frac{x^3-4x^2+2}{(x^2+1)(x^2+2)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2}$

12. Find the vertex, focus and directrix of the parabola. Write the vertex and focus as ordered pairs and write the directrix as an equation.

$(x+3)^2+8(y-4)=0$

$(x-h)^2=4p(y-k)$

rewrite it like this

$(x+3)^2=-8(y-4)$

vertical parabola

$(h, k+p)$

$(-3, 4+(-2))$ or $(-3,2)$

vertex $=-3,4$

p is negative so parabola opens down

$\frac{-8}{4}=\frac{4p}{4}$

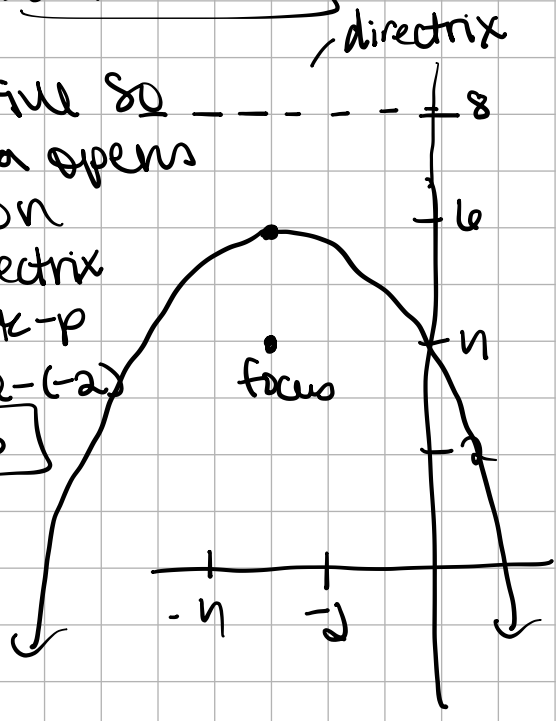
$p=-2$

directrix

$y=k-p$

$y=4-(-2)$

$y=6$



13. The point $(1,-2)$ lies on a circle whose center is at $(-3,-5)$. Write the standard form for the equation of the circle

$x^2+y^2=1$

$(x-h)^2+(y-k)^2=r^2$

$(1-(-3))^2+(-2-(-5))^2=r^2$

$-2+5$

$(4)^2+(3)^2=r^2$

$16+9=r^2$

$25=r^2$

$r=5$

$(x+3)^2+(y+5)^2=25$

11. $x^2+y^2+4x-4y-17=0$ is an equation of a circle. Find the standard form of the equation

$x^2+y^2+4x-4y-17=0$

$+17 \quad +17$

$(x^2+4x+\underline{4}\,)+(y^2-4y+\underline{4}\,)=17+4+4$

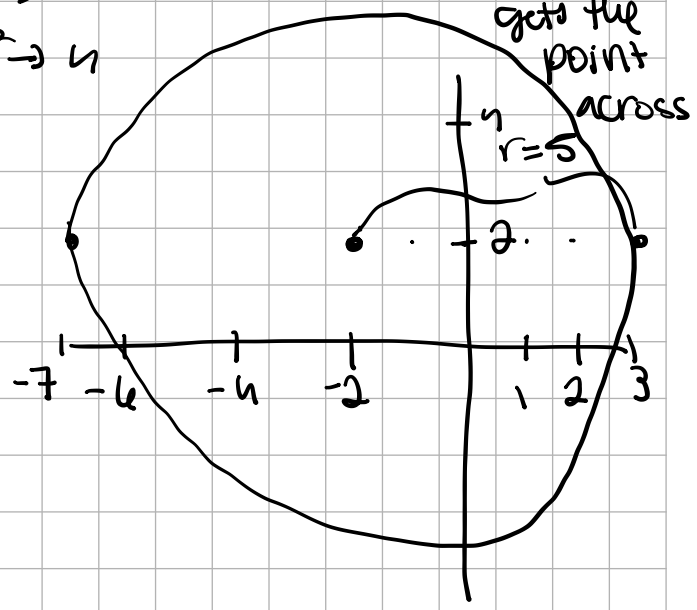
$4/2 \to 2^2=4 \qquad -4/2 \to -2^2 \to 4$

$\boxed{(x+2)^2+(y-2)^2=25}$

Center: $-2, 2$

radius: $r=5$

rough but it gets the point across



15. $4x^2+y^2-8x+4y-8=0$ Find the standard form of the ellipse

$4x^2+y^2-8x+4y-8=0$

$+8 \quad +8$

factor out 4

$4(x^2-2x+\underline{1}\,)+(y^2+4y+\underline{4}\,)=8+4+4$

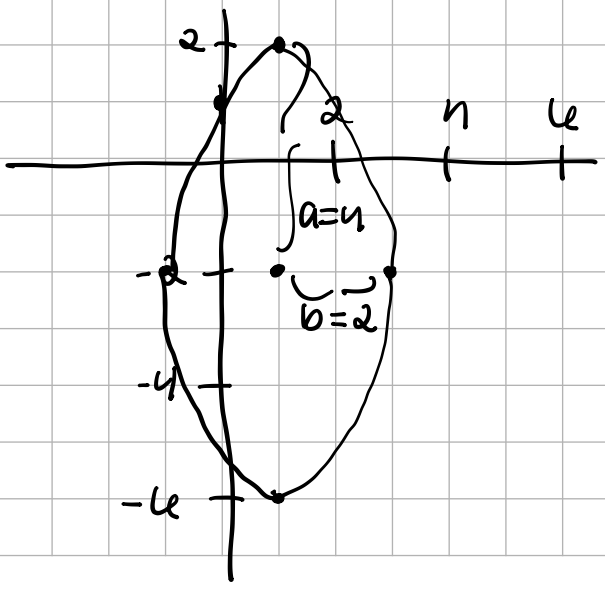
$-2/1 \to -1^2=1 \qquad 4/2 \to 2^2=4$

$\frac{4(x-1)^2+(y+2)^2=16}{16}$ ← needs to equal 1

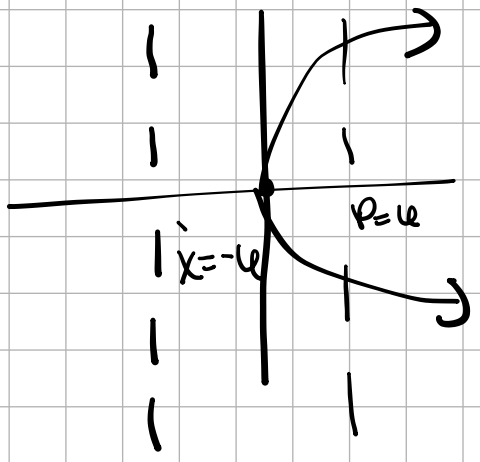
$\boxed{\frac{(x-1)^2}{4}+\frac{(y+2)^2}{16-4}=1}$

b

vertex @ $1, -2$



16. find the standard form with the directrix at $x=-6$



$p=|d|$

$p=6$

$y^2=4px$

$y^2=4(6)x$

$\boxed{y^2=24x}$

17. Write the equation in standard form (what kind of conic is this?)

$x^2-4y^2-2x-40y-103=0$

$x^2-4y^2-2x-40y=103$

-1 factored out then

$(x^2-2x+1)-4(y^2+10y+25)=103+1-100$

$-2/2\to-1^2=1$ $\quad +10/2\to-5^2=25$

$\frac{(x-1)^2}{4}-\frac{4(y-5)^2}{4}=\frac{4}{4}$

$\boxed{\frac{(x-1)^2}{4}-\frac{(y+5)^2}{1}=1}$

horizontal (looks vertical)

I graphed the last question on the next page

asymptotes $y=\pm\frac{1}{2}x$ $\quad \frac{b}{a}$ - not squared

* these numbers depend on whether a or b is with x or y.
this is when x is with a

vertex = (1, -5)

asymptotes $\pm \frac{1}{2}x$