Name: Answer key Date: 4/20/26

Session 15

Dot product reminders!!!

If you have $\vec{a} = \langle a_1, a_2 \rangle$ and $\vec{b} = \langle b_1, b_2 \rangle$,

then your formula is

$$\vec{a} \cdot \vec{b} = (a_1 b_1) + (a_2 b_2)$$

your answer should just be a number, not a vector!

If you have an angle between $\vec{a}$ and $\vec{b}$, then

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta \quad \text{or} \quad \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

If $\vec{a}$ and $\vec{b}$ are perpendicular (orthogonal), then and only then,

$$\vec{a} \cdot \vec{b} = 0$$

↑ regular zero

Properties of dot product

$$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u} \qquad (c\vec{u})\vec{v} = c(\vec{u} \cdot \vec{v}) = \vec{u}(c\vec{v})$$

$$(\vec{u} + \vec{v})\vec{w} = \vec{u} \cdot \vec{w} + \vec{v} \cdot \vec{w} \qquad |\vec{u}|^2 = \vec{u} \cdot \vec{u}$$

What is the definition of a vector?

a quantity that has both direction and magnitude

Useful properties to know!
(as you practice, these will become less scary, I promise!!)

$\vec{u}+\vec{v}=\vec{v}+\vec{u}$

$\vec{u}+(\vec{v}+\vec{w})=(\vec{u}+\vec{v})+\vec{w}$

$\vec{u}+\vec{0}=\vec{u}$

$\vec{u}+(-\vec{u})=\vec{0}$

$|c\vec{u}|=c|\vec{u}|$

$C(\vec{u}+\vec{v})+c\vec{u}+c\vec{v}$

$(c+d)\vec{u}=c\vec{u}+d\vec{u}$

$(cd)\vec{u}=c(d\vec{u})=d(c\vec{u})$

$1\cdot\vec{u}=\vec{u}$

$0\cdot\vec{u}=\vec{0}$

$c\cdot\vec{0}=\vec{0}$

Some more helpful stuff to know about vectors!!

$|\vec{a}|=\sqrt{(a_1)^2+(a_2)^2}$

1. Determine the dot product of the two vectors and the angle between them

$\vec{a} = 3i + 4j$ $\quad\quad$ $\vec{b} = 5i - 2j$

$\vec{a} = \langle 3, 4 \rangle$ $\quad\quad$ $\vec{b} = \langle 5, -2 \rangle$

$\vec{a} \cdot \vec{b} = (3 \cdot 5) + (4 \cdot -2)$

$15 + (-8)$

7

$|\vec{a}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$|\vec{b}| = \sqrt{5^2 + (-2)^2} = \sqrt{29} = \sqrt{29}$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} \rightarrow \cos\theta = \dfrac{7}{5 \cdot \sqrt{29}} \rightarrow \cos\theta = 0.25991734$

$\cos^{-1}(0.25991734) = \theta$

$\theta = 74.93°$ ← make sure your calculator is in deg. mode!

$\vec{a} = \langle 15, 6, -10 \rangle$ $\quad\quad$ $\vec{b} = \langle 2, 5, -3 \rangle$

$\vec{a} \cdot \vec{b} = (15 \cdot 2) + (6 \cdot 5) + (-10 \cdot -3)$

$30 + 30 + 30$

$\vec{a} \cdot \vec{b} = 90$

$|\vec{a}| = \sqrt{15^2 + 6^2 + (-10)^2} = 19$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$|\vec{b}| = \sqrt{2^2 + 5^2 + (-3)^2}$

$\sqrt{38}$

$\cos\theta = \dfrac{90}{19 \cdot \sqrt{38}} \rightarrow \cos\theta = 0.768417$

$\cos^{-1}(0.768417) = \theta$

$\theta = 39.79$

2. A forklift moves 34 meters, carrying a 1032 N box across the floor. How much work is being done by the forklift?

$W = \vec{F} \cdot \vec{d}$

$\vec{F} = \langle 1032, 0 \rangle$

$\vec{d} = \langle 34, 0 \rangle$

We have no force or distance working in the y-direction

$\vec{F} \cdot \vec{d} = (1032 \cdot 34) + (0 \cdot 0)$

$\downarrow$

$35{,}088\ J$

J is N·m but all are used to lb-ft Still force · distance, just different quantities

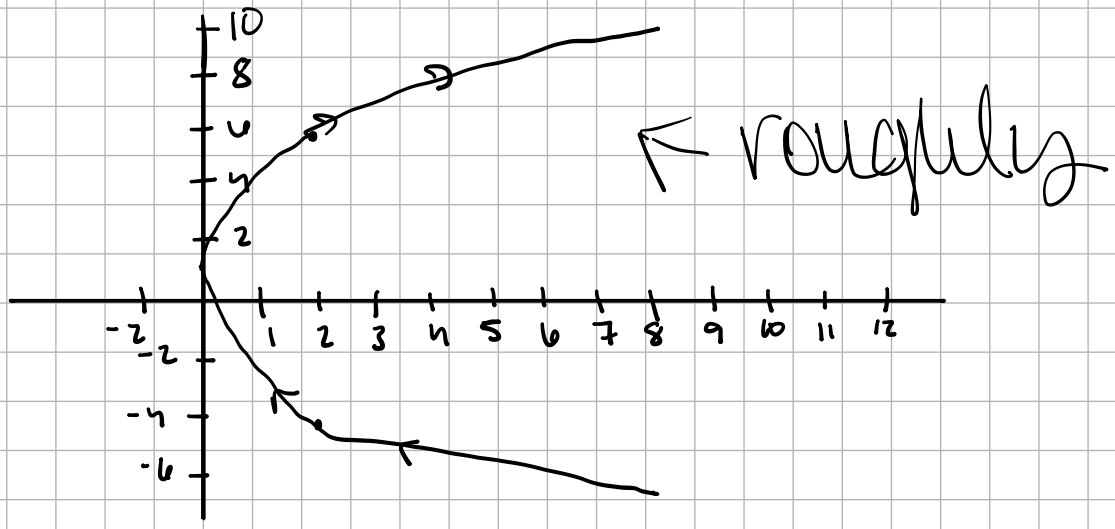
3. Create a table for the following parametric equations! $-2 \leq t \leq 2$

$x = 2t^2$

$y = 5t + 1$

t	x	y
-2	8	-9
-1	2	-4
0	0	1
1	2	6
2	8	11

Now, graph the function. Determine its direction and draw arrows to show it!



← roughly

Finally, eliminate the parameter in the following parametric equations

(it doesn't matter if its x= or y=, whatever is easier for you to solve for!)

$x = 2t^2$

$y = 5t + 1$

I'm going to solve for y

$x = 2t^2$

$y = 5t + 1$

$y = 5t + 1$
-1

$y - 1 = 5t$

$\frac{y-1}{5} = t$

$x = 2\left(\frac{y-1}{5}\right)^2$

$x = 2\left(\frac{y^2-2y+1}{25}\right)$

$\frac{(y-1)^2}{5^2}$ y^2-2y+1

$\rightarrow \boxed{x = \frac{2y^2-4y+2}{25}}$

4. A dog pulls a sled with a force of 250 N for a distance of 75 meters. How much work did the dog do on the sled?

$W = F \times d$

$F = 250N$

$d = 75m$

$W = 18,750J$ work

or

18.75 kJ of work

both work